Year 10 Maths Sceme of Work

Edexcel Units 6-12 GCSE Textbook

Term	*Lessons*	*Key Areas*	
Autumn 1	20	• Investigate features of straight line graphs • Explore graphs of quadratic functions • Explore graphs of other standard non-linear functions • Create and use graphs of non-standard functions • Solve kinematic problems • Understand and use set notation • Solve inequalities • Represent inequalities on a graph • Explore exponential graphs • Create and use graphs of non-standard functions • Investigate gradients of graphs • Find and interpret areas under graphs • Investigate features of quadratic graphs • Area and Perimeter of 2D and 3D shapes • Explore transformations of 2D shapes	
Autumn 2	18		
Spring 1	18		
Spring 2	15		
Summer 1	15		
Summer 2	15		
Total:	101		

Autumn 1: UNIT 6 – STRAIGHT LINE GRAPHS | GCSE EDEXCEL HIGHER TEXTBOOK | 10 lessons

Key concepts (GCSE subject content statements)

The Big Picture: Algebra progression map

- identify and interpret gradients and intercepts of linear functions algebraically
- use the form *y = mx + c* to identify parallel lines
- find the equation of the line through two given points, or through one point with a given gradient
- interpret the gradient of a straight line graph as a rate of change
- recognise, sketch and interpret graphs of quadratic functions
- recognise, sketch and interpret graphs of simple cubic functions and the reciprocal function $y = 1/x$ with $x \neq 0$
- plot and interpret graphs (including reciprocal graphs) and graphs of non-standard functions in real contexts, to find approximate solutions to problems such as simple kinematic problems involving distance, speed and acceleration

Return to overview

Possible themes

- Investigate features of straight line graphs
- Explore graphs of quadratic functions
- Explore graphs of other standard non-linear functions
- Create and use graphs of non-standard functions
- Solve kinematic problems

Possible key learning points

- Identify and interpret gradients of linear functions algebraically
- Identify and interpret intercepts of linear functions algebraically
- Use the form y = mx + c to identify parallel lines
- Find the equation of a line through one point with a given gradient
- Find the equation of a line through two given points
- Interpret the gradient of a straight line graph as a rate of change
- Plot graphs of quadratic functions
- Plot graphs of cubic functions
- Plot graphs of reciprocal functions
- Recognise and sketch the graphs of quadratic functions
- Interpret the graphs of quadratic functions
- Recognise and sketch the graphs of cubic functions
- Interpret the graphs of cubic functions
- Recognise and sketch the graphs of reciprocal functions
- Interpret the graphs of reciprocal functions
- Plot and interpret graphs of non-standard functions in real contexts
- Find approximate solutions to kinematic problems involving distance, speed and acceleration

Prerequisites

- Plot straight-line graphs
- Interpret gradients and intercepts of linear functions graphically and algebraically
- Recognise, sketch and interpret graphs of linear functions
- Recognise graphs of simple quadratic functions
- Plot and interpret graphs of kinematic problems involving distance and speed

Mathematical language

Function, equation
Quadratic, cubic, reciprocal
Gradient, y-intercept, x-intercept, root
Sketch, plot
Kinematic
Speed, distance, time
Acceleration, deceleration
Linear, non-linear
Parabola, Asymptote
Rate of change

Notation

y = mx + c

Pedagogical notes

This unit builds on the graphs of linear functions and simple quadratic functions work from Stage 8. Where possible, students should be encouraged to plot linear graphs efficiently by using knowledge of the *y*-intercept and the gradient.

NCETM: Glossary

Common approaches

'Monter' and 'commencer' are shared as the reason for 'm' and 'c' in y = mx + c and links to y = ax + b.

All student use dynamic graphing software to explore graphs

Reasoning opportunities and probing questions

- Convince me the lines y = 3 + 2x, y – 2x = 7, 2x + 6 = y and 8 + y – 2x = 0 are parallel to each other.
- What is the same and what is different: y = x, $y = x^2$, $y = x^3$ and y=1/x ?
- Show me a sketch of a quadratic (cubic, reciprocal) graph. And another. And another …
- Sketch a distance/time graph of your journey to school. What is the same and what is different with the graph of a classmate?

Suggested Activities

KM: Screenshot challenge
KM: Stick on the Maths: Quadratic and cubic functions
KM: Stick on the Maths: Algebraic Graphs
KM: Stick on the Maths: Quadratic and cubic functions
NRICH: Diamond Collector
NRICH: Fill me up
NRICH: What's that graph?
NRICH: Speed-time at the Olympics
NRICH: Exploring Quadratic Mappings
NRICH: Minus One Two Three
Learning review KM: 9M4 BAM Task, 9M6 BAM Task

Possible misconceptions

- Some students do not rearrange the equation of a straight line to find the gradient of a straight line. For example, they think that the line y – 2x = 6 has a gradient of -2.
- Some students may think that gradient = (change in x) / (change in y) when trying to equation of a line through two given points.
- Some students may incorrectly square negative values of x when plotting graphs of quadratic functions.
- Some students think that the horizontal section of a distance time graph means an object is travelling at constant speed.
- Some students think that a section of a distance time graph with negative gradient means an object is travelling backwards or downhill.

Autumn 1: UNIT 6 – STRAIGHT LINE GRAPHS | GCSE EDEXCEL HIGHER TEXTBOOK | 5 lessons

Key concepts (GCSE subject content statements)

The Big Picture: Algebra progression map

- solve linear inequalities in two variables
- represent the solution set to an inequality using set notation and on a graph

Return to overview

Possible themes

- Understand and use set notation
- Solve inequalities
- Represent inequalities on a graph

Possible key learning points

- State the (simple) inequality represented by a shaded region on a graph
- Construct and shade a graph to show a linear inequality of the form y > ax + b, y < ax + b, y ≥ ax + b or y ≤ ax + b
- Construct and shade a graph to show a linear inequality in two variables stated implicitly
- Construct and shade a graph to represent a set of linear inequalities in two variables
- Find the set of integer coordinates that are solutions to a set of inequalities in two variables
- Use set notation to represent the solution set to an inequality

Prerequisites

- Understand the meaning of the four inequality symbols
- Find the set of integers that are solutions to an inequality
- Use set notation to list a set of integers
- Use a formal method to solve an inequality in one variable
- Plot graphs of linear functions stated explicitly
- Plot graphs of linear functions stated implicitly

Mathematical language

(Linear) inequality
Variable
Manipulate
Solve
Solution set
Integer
Set notation
Region

Notation

The inequality symbols: < (less than), > (greater than), ≤ (less than or equal to), ≥ (more than or equal to)
A graph to represent solutions to inequalities in two variables. A dotted line represents a boundary that is not included. A solid line represents a boundary that is included.
Set notation; e.g. {-2, -1, 0, 1, 2, 3, 4}

Pedagogical notes

Pupils have explored the meaning of an inequality and solved linear inequalities in one variable in Stage 9. This unit focuses on solving linear equalities in two variables, representing the solution set using set notation and on a graph Therefore, it is important that pupils can plot the graphs of linear functions, including x = a and y = b.
NCETM: Departmental workshops: Inequalities
NCETM: Glossary

Common approaches

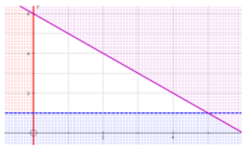
All students experience the use of dynamic graphing software, such as Autograph, to represent the solution sets of inequalities in two variables. Students are taught to manipulate algebraically rather than be taught 'tricks'. For example, in the case of -2x > 8, students should not be taught to flip the inequality when dividing by -2. They should be taught to add 2x to both sides. Many students will later generalise themselves. Note that with examples such as 5 < 1 – 4x < 21, subtracting 1 from all three parts, and then adding 4x, results in 4 + 4x < 0 < 20 + 4x. This could be broken down into two inequalities to discover that x < -1 and x > -5, so -5 < x < -1. The 'trick' results in the more unconventional solution -1 > x > -5.

Reasoning opportunities and probing questions

- Show me a pair of integers that satisfy x + 2y < 6. And another. And another …
- Convince me that the set of inequalities x > 0, y > 0 and x + y < 2 has no positive integer solutions.
- Convince me that the set of inequalities x ≥ 0, y > 0 and x + 2y < 6 has 6 pairs of positive integer solutions.
- What is wrong with this statement? How can you correct it?

The unshaded region represents the solution set for the inequalities:

x < 1, y ≥ 0 and x + y > 6



Suggested activities

KM: Linear programming with Lego
KM: Linear programming (Autograph)
KM: Stick on the Maths 8: Inequalities
KM: Convinced?: Inequalities in two variables
NRICH: Which is bigger?
Hwb: How do we know?
MAP: Defining regions using inequalities
CIMT: Inequalities

Learning review

GLOWMaths/JustMaths: Sample Questions Both Tiers
GLOWMaths/JustMaths: Sample Questions Higher Tiers

Possible misconceptions

- Some pupils may think that it is possible to multiply or divide both sides of an inequality by a negative number with no impact on the inequality (e.g. if -2x > 12 then x > -6)
- Some pupils may think that strict inequalities, such as y < 2x + 3, are represented by a solid, rather than dashed, line on a graph
- Some pupils may shade the incorrect region

Autumn 1: UNIT 6 – REAL LIFE AND OTHER GRAPHS | GCSE EDEXCEL HIGHER TEXTBOOK | 5 lessons

Key concepts (GCSE subject content statements)

The Big Picture: Algebra progression map

- plot and interpret graphs (including exponential graphs) and graphs of non-standard functions in real contexts, to find approximate solutions to problems such as simple kinematic problems involving distance, speed and acceleration
- calculate or estimate gradients of graphs and areas under graphs (including quadratic and other non-linear graphs), and interpret results in cases such as distance-time graphs, velocity-time graphs and graphs in financial contexts
- interpret the gradient at a point on a curve as the instantaneous rate of change
- identify and interpret roots, intercepts, turning points of quadratic functions graphically

Return to overview

Possible themes

- Explore exponential graphs
- Create and use graphs of non-standard functions
- Investigate gradients of graphs
- Find and interpret areas under graphs
- Investigate features of quadratic graphs

Possible key learning points

- Recognise, plot and interpret exponential graphs
- Plot graphs of non-standard functions
- Use graphs of non-standard functions to solve simple kinematic problems
- Recognise that the gradient of a curve is not constant
- Know that the gradient of a curve is the gradient of the tangent at that point
- Calculate the gradient at a point on a curve
- Interpret the gradient at a point on a curve as the instantaneous rate of change
- Interpret the gradient of a chord as an average rate of change
- Solve problems involving the gradients of graphs in context
- Calculate an estimate for the area under a graph, including the area under a speed-time graph as distance
- Solve problems involving the area under graphs in context
- Identify and interpret roots, intercepts and turning points of quadratic functions graphically

Prerequisites

- Plot graphs of linear, quadratic, cubic and reciprocal functions
- Interpret the gradient of a straight line graph as a rate of change
- Plot and interpret graphs of kinematic problems involving distance and speed

Mathematical language

Function, equation
Linear, non-linear
Quadratic, cubic, reciprocal, exponential
Parabola, Asymptote
Gradient, y-intercept, x-intercept, root
Rate of change
Sketch, plot
Kinematic
Speed, distance, time
Acceleration, deceleration

Notation

$y = mx + c$

Pedagogical notes

Pupils have met plotting graphs of non-standard functions and using graphs of non-standard functions to solve simple kinematic problems in Stage 9. This unit explores and deepens pupils' understanding of these concepts. However, they do not explicitly plot graphs of exponential functions until Stage 11.
This unit also introduces the concept of gradient as an instantaneous change. Drawing tangents at different points on quadratic/cubic graphs and calculating an estimate of their gradients is a very powerful activity for pupils to appreciate the gradient can change.
NCETM: Glossary

Common approaches

All pupils use dynamic graphing software, e.g. Autograph and the 'gradient value/function', to explore graphs

Reasoning opportunities and probing questions

- Show me a sketch of an exponential graph. And another. And another …
- What is the same and what is different: $y = x^2$, $y = 2^x$, $y=1/2x$ and $y = (1/2)^x$?
- Always/Sometimes/Never: The gradient of a function is constant.
- Sketch a speed/time graph of your journey to school. What is the same and what is different with the graph of a classmate?

Suggested activities

KM: Autograph: Pre-Calculus Activity
KM: Autograph: The numerical gradient
NRICH: What's that graph?
Hwb: The 100m race
MAP: Representing functions of everyday situations
ILIM: Interpreting Distance Time Graphs
GCSE: Subject Knowledge Check - Tangents to a curve and Areas under a curve
Learning review
GLOWMaths/JustMaths: Sample Questions Both Tiers
GLOWMaths/JustMaths: Sample Questions Higher Tiers
KM: 10M8 BAM Task

Possible misconceptions

- Some pupils may think the graphs of all quadratic functions intercept the x-axis in one or two places.
- Some pupils may think that gradient has the same value for all points for all functions
- Some pupils may join the graph of $y = a^x$ (a>1) to the x-axis
- Some pupils think that the horizontal section of a distance time graph means an object is travelling at constant speed.
- Some pupils think that a section of a distance time graph with negative gradient means an object is travelling backwards or downhill.

Autumn 2: UNIT 7 – PERIMETER, AREA, VOLUME	GCSE EDEXCEL HIGHER TEXTBOOK	3 lessons

Key concepts (GCSE subject content statements)

The Big Picture: Measurement and mensuration progression map

- identify and apply circle definitions and properties, including: tangent, arc, sector and segment
- calculate arc lengths, angles and areas of sectors of circles
- *calculate surface area of right prisms (including cylinders)*
- calculate exactly with multiples of π
- know the formulae for: Pythagoras' theorem, $a^2 + b^2 = c^2$, and apply it to find lengths in right-angled triangles in two dimensional figures

Return to overview

Possible themes

- Solve problems involving arcs and sectors
- Solve problems involving prisms
- Investigate right-angled triangles
- Solve problems involving Pythagoras' theorem

Possible key learning points

- Know circle definitions and properties, including: tangent, arc, sector and segment
- Calculate the arc length of a sector, including calculating exactly with multiples of π
- Calculate the area of a sector, including calculating exactly with multiples of π
- Calculate the angle of a sector when the arc length and radius are known
- Calculate the surface area of a right prism
- Calculate the surface area of a cylinder, including calculating exactly with multiples of π
- Know and use Pythagoras' theorem
- Calculate the hypotenuse of a right-angled triangle using Pythagoras' theorem in two dimensional figures
- Calculate one of the shorter sides in a right-angled triangle using Pythagoras' theorem in two dimensional figures
- Solve problems using Pythagoras' theorem in two dimensional figures

Prerequisites

- Know and use the number π
- Know and use the formula for area and circumference of a circle
- Know how to use formulae to find the area of rectangles, parallelograms, triangles and trapezia
- Know how to find the area of compound shapes

Mathematical language

Circle, Pi
Radius, diameter, chord, circumference, arc, tangent, sector, segment
(Right) prism, cylinder
Cross-section
Hypotenuse
Pythagoras' theorem

Notation

π
Abbreviations of units in the metric system: km, m, cm, mm, mm^2, cm^2, m^2, km^2, mm^3, cm^3, km^3

Pedagogical notes

This unit builds on the area and circle work form Stages 7 and 8. Students will need to be reminded of the key formula, in particular the importance of the perpendicular height when calculating areas and the correct use of πr^2. Note: some students may only find the area of the three 'distinct' faces when finding surface area.
Students must experience right-angled triangles in different orientations to appreciate the hypotenuse is always opposite the right angle.
NCETM: Glossary
Common approaches
Students visualize and write down the shapes of all the faces of a prism before calculating the surface area. Every classroom has a set of area posters on the wall.
Pythagoras' theorem is stated as 'the square of the hypotenuse is equal to the sum of the squares of the other two sides' not just $a^2 + b^2 = c^2$.

Reasoning opportunities and probing questions

- Show me a sector with area 25π. And another. And another …
- Always/ Sometimes/ Never: The value of the volume of a prism is less than the value of the surface area of a prism.
- Always/ Sometimes/ Never: If $a^2 + b^2 = c^2$, a triangle with sides a, b and c is right angled.
- Kenny thinks it is possible to use Pythagoras' theorem to find the height of isosceles triangles that are not right- angled. Do you agree with Kenny? Explain your answer.
- Convince me the hypotenuse can be represented as a horizontal line.

Suggested activities

KM: The language of circles
KM: One old Greek (geometrical derivation of Pythagoras' theorem. This is explored further in the next unit)
KM: Stick on the Maths: Pythagoras' Theorem
KM: Stick on the Maths: Right Prisms
NRICH: Curvy Areas
NRICH: Changing Areas, Changing Volumes

Learning review
KM: 9M10 BAM Task, 9M11 BAM Task

Possible misconceptions

- Some students will work out $(\pi \times r)^2$ when finding the area of a circle
- Some students may use the sloping height when finding cross-sectional areas that are parallelograms, triangles or trapezia
- Some students may confuse the concepts of surface area and volume
- Some students may use Pythagoras' theorem as though the missing side is always the hypotenuse
- Some students may not include the lengths of the radii when calculating the perimeter of an sector

Autumn 2: UNIT 7 – PERIMITER, AREA & VOLUME | GCSE EDEXCEL HIGHER TEXTBOOK | 3 lessons

Key concepts (GCSE subject content statements)

The Big Picture: Measurement and mensuration progression map

- calculate surface area and volume of spheres, pyramids, cones and composite solids
- apply the concepts of congruence and similarity, including the relationships between length, areas and volumes in similar figures

Return to overview

Possible themes

- Calculate surface areas of solids
- Calculate volumes of solids
- Solve problems involving enlargement and 3D shapes

Possible key learning points

- Use Pythagoras' theorem to find lengths in a pyramid or cone
- Find the surface area of spheres, cones and pyramids
- Find the volume of spheres, cones and pyramids
- Identify how to find the volume or surface area of a composite solid
- Solve practical problems involving the surface area of solids
- Solve practical problems involving the volume of solids
- Understand the implications of enlargement on area
- Understand the implications of enlargement on volume
- Move freely between scale factors for length, area and volume
- Solve practical problems involving length, area and volume in similar figures

Prerequisites

- Calculate exactly with multiples of π
- Know and use the formula for area and circumference of a circle
- Know how to use formulae to find the area of rectangles, parallelograms, triangles, trapezia, circles, sectors and
- Know how to find the area of compound shapes
- Know how to find the surface area of a right prism and a cylinder
- Calculate the surface area of a right prism and a cylinder
- Carry out an enlargement
- Find the scale factor of a given enlargement
- Use Pythagoras' theorem to find missing lengths in right-angled triangles

Mathematical language

(Composite) solid
Sphere, Pyramid, Cone
Perpendicular (height), (slant height)
Surface area
Volume
Congruent, congruence
Similarity, similar shapes, similar figures
Enlarge, enlargement
Scale factor

Notation

π
Abbreviations of units in the metric system: km, m, cm, mm, mm^2, cm^2, m^2, km^2, mm^3, cm^3, km^3

Pedagogical notes

Pupils have previously learnt how to find the surface area of right prisms and cylinders in Stage 9. This unit focuses on finding the volume and surface areas of cones, spheres and pyramids.
Pupils also explore congruence and similarity - the use of proportion tables can be helpful to find the multiplier when solving similarity problems such as:

	Shape A	Shape B
Known lengths	6	9
Missing lengths	10	15

→ × 1.5 →

NCETM: Glossary

Common approaches

Pupils explore the surface area of spheres using oranges (https://www.youtube.com/watch?v=cAxHYFRx1Fs)
Pupils explore volumes of pyramids by making nets of pyramids and prisms with the same polygonal base and using sand or sugar to compare volumes.

Reasoning opportunities and probing questions

- Always/ Sometimes/ Never: The value of the volume of a pyramid is less than the value of the surface area of a pyramid.
- Always/ Sometimes/ Never: The value of the volume of a sphere is less than the value of the surface area of a sphere.
- Convince me that the volume of a pyramid = $^1/_3$ × A × h
- Convince me that 1 m^3 = 1 000 000 cm^3

Suggested activities

KM: Stick on the Maths 8: Congruence and Similarity
KM: Convinced? Congruence and Similarity
NRICH: Surface Area and Volume and Nicely Similar
Hwb: Summerhouse and Radiators
OCR: Congruence Check In and Similarity Check In

Learning review

GLOWMaths/JustMaths: Sample Questions Both Tiers
GLOWMaths/JustMaths: Sample Questions Higher Tiers
KM: 10M11 BAM Task

Possible misconceptions

- Some pupils will work out $4/3 \times (\pi \times r)^3$ when finding the volume of a sphere.
- Some pupils may confuse the concepts of surface area and volume
- Some pupils will work out $4 \times (\pi \times r)^2$ when finding the surface area of a sphere.
- Some pupils may think the volume of a pyramid = ½ × A × h

Key concepts (GCSE subject content statements)

The Big Picture: Properties of Shape progression map

- use the standard ruler and compass constructions (perpendicular bisector of a line segment, constructing a perpendicular to a given line from/at a given point, bisecting a given angle)
- use these to construct given figures and solve loci problems; know that the perpendicular distance from a point to a line is the shortest distance to the line
- construct plans and elevations of 3D shapes

Return to overview

Possible themes

- Know standard mathematical constructions
- Apply standard mathematical constructions
- Explore ways of representing 3D shapes

Possible key learning points

- Use ruler and compasses to construct the perpendicular bisector of a line segment
- Use ruler and compasses to bisect an angle
- Use a ruler and compasses to construct a perpendicular to a line from a point and at a point
- Know how to construct the locus of points a fixed distance from a point and from a line
- Solve simple problems involving loci
- Combine techniques to solve more complex loci problems
- Choose techniques to construct 2D shapes; e.g. rhombus
- Construct a shape from its plans and elevations
- Construct the plan and elevations of a given shape

Prerequisites

- Measure distances to the nearest millimetre
- Create and interpret scale diagrams
- Use compasses to draw circles
- Interpret plan and elevations

Mathematical language

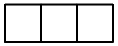
Compasses
Arc
Line segment
Perpendicular
Bisect
Perpendicular bisector
Locus, Loci
Plan
Elevation

Pedagogical notes

Ensure that students always leave their construction arcs visible. Arcs must be 'clean'; i.e. smooth, single arcs with a sharp pencil.
NCETM: Departmental workshops: Constructions
NCETM: Departmental workshops: Loci
NCETM: Glossary

Common approaches
All students should experience using dynamic software (e.g. Autograph) to explore standard mathematical constructions (perpendicular bisector and angle bisector).

Reasoning opportunities and probing questions

- (Given a single point marked on the board) show me a point 30 cm away from this point. And another. And another …
- Provide shapes made from some cubes in certain orientations. Challenge students to construct the plans and elevations. Do groups agree?
- If this is the plan: 
  show me a possible 3D shape. And another. And another.
- Demonstrate how to create the perpendicular bisector (or other constructions). Challenge students to write a set of instructions for carrying out the construction. Follow these instructions very precisely (being awkward if possible; e.g. changing radius of compasses). Do the instructions work?
- Give students the equipment to create standard constructions and challenge them to create a right angle / bisect an angle

Suggested activities

KM: Construction instruction
KM: Construction challenges
KM: Napoleonic challenge
KM: Circumcentre etcetera
KM: Locus hocus pocus
KM: The perpendicular bisector
KM: Topple
KM: Gilbert goat
KM: An elevated position
KM: Solid problems (plans and elevations)
KM: Isometric interpretation (plans and elevations)

Learning review
KM: 9M8 BAM Task

Possible misconceptions

- When constructing the bisector of an angle some students may think that the intersecting arcs need to be drawn from the ends of the two lines that make the angle.
- When constructing a locus such as the set of points a fixed distance from the perimeter of a rectangle, some students may not interpret the corner as a point (which therefore requires an arc as part of the locus)
- The north elevation is the view of a shape from the north (the north face of the shape), not the view of the shape while facing north.

Autumn 2: UNIT 8 – TRANSFORMATIONS	GCSE EDEXCEL HIGHER TEXTBOOK	3 lessons

Key concepts (GCSE subject content statements)

The Big Picture: Position and direction progression map

- identify, describe and construct similar shapes, including on coordinate axes, by considering enlargement (including fractional scale factors)
- make links *between* similarity and scale factors
- describe the changes and invariance achieved by combinations of rotations, reflections and translations

Return to overview

Possible themes	Possible key learning points
• Explore enlargement of 2D shapes • Investigate the transformation of 2D shapes	• Use the centre and scale factor to carry out an enlargement of a 2D shape with a fractional scale factor • Find the scale factor of an enlargement with fractional scale factor • Find the centre of an enlargement with fractional scale factor • Solve problems involving similarity • Perform a sequence of transformations on a 2D shape • Find and describe a single transformation given two congruent 2D shapes

Prerequisites	Mathematical language	Pedagogical notes
• Use the centre and scale factor to carry out an enlargement of a 2D shape with a positive integer scale factor • Use the concept of scaling in diagrams • Carry out reflection, rotations and translations of 2D shapes	Perpendicular bisector Scale Factor Similar Congruent Invariance Transformation Rotation Reflection Translation Enlargement	Pupils have identified, described and constructed congruent shapes using rotation, reflection and translation in Stage 7. They have also identified, described and constructed similar shapes using enlargement in Stage 8 and experienced enlarging shapes using positive integer scale factors in Stage 9. NCETM: Glossary **Common approaches** *All pupils should experience using dynamic software (e.g. Autograph) to explore enlargements using fractional scale factors*

Reasoning opportunities and probing questions	Suggested activities	Possible misconceptions
• Show me a pair of similar shapes. And another. And another … • Always/ Sometimes/ Never: The resulting image of an enlargement is larger than the original object • Kenny thinks rotating an object 90° about the origin followed by a reflection in the y-axis has the same effect as reflecting an object in the y-axis followed by a rotation 90° about the origin. Do you agree with Kenny? Explain your answer.	KM: Enlargement 2 KM: Stick on the Maths SSM3: Enlargement (fractional scale factor) KM: Stick on the Maths SSM1: Congruence and similarity NRICH: Growing Rectangles **Learning review** GLOWMaths/JustMaths: Sample Questions Both Tiers GLOWMaths/JustMaths: Sample Questions Higher Tiers	• Some pupils may think that the resulting image of an enlargement has to be larger than the original object. • Some pupils may think that the order of transforming an object does not have an effect on the size and position of the final image. • Some pupils may link scale factors and similarity using an additive, rather than multiplicative, relationship.

Autumn 2: UNIT 8 – TRANSFORMATIONS | GCSE EDEXCEL HIGHER TEXTBOOK | 3 lessons

Key concepts (GCSE subject content statements)

The Big Picture: Position and direction progression map

- identify, describe and construct similar shapes, including on coordinate axes, by considering enlargement (including negative scale factors)

Return to overview

Possible themes	Possible key learning points
• Explore enlargement of 2D shapes **Bring on the Maths: GCSE Higher Shape** Manipulating shapes I: #5	• Use the centre and scale factor to carry out an enlargement of a 2D shape with a negative scale factor • Find the scale factor of an enlargement with negative scale factor • Find the centre of an enlargement with negative scale factor

Prerequisites	Mathematical language	Pedagogical notes
• Use the centre and scale factor to carry out an enlargement of a 2D shape with a positive scale factor	Scale Factor Similar Transformation Enlargement	Pupils have identified, described and constructed similar shapes using enlargement in Stage 8 and experienced enlarging shapes using positive integer scale factors in Stage 9. Stage 10 included enlargement using a fractional scale factor. NCETM: Glossary **Common approaches** *All pupils should experience using dynamic software (e.g. Autograph) to explore enlargements using negative scale factors*

Reasoning opportunities and probing questions	Suggested activities	Possible misconceptions
• Always/ Sometimes/ Never: The resulting image of an enlargement is larger than the original object	KM: Enlargement 3 NRICH: Transformation game **Learning review** GLOWMaths/JustMaths: Sample Questions Higher Tiers	• Some pupils may think that the resulting image of an enlargement has to be larger than the original object. • Some pupils may link scale factors and similarity using an additive, rather than multiplicative, relationship.

Autumn 2 – Assessment | 3 lessons

- 1.5 hourS non calculator Foundation Paper
- Self-assessment sheets completed
- Review and self-assessment of performance stuck into books

Key concepts (GCSE subject content statements)

The Big Picture: Algebra progression map

- solve, in simple cases, two linear simultaneous equations in two variables algebraically
- derive an equation (or two simultaneous equations), solve the equation(s) and interpret the solution
- find approximate solutions to simultaneous equations using a graph

Return to overview

Possible themes	Possible key learning points
• Solve simultaneous equations • Use graphs to solve equations • Solve problems involving simultaneous equations	• Understand that there are an infinite number of solutions to the equation ax + by = c ($a \neq 0$, $b \neq 0$) • Find approximate solutions to simultaneous equations using a graph • Solve two linear simultaneous equations in two variables in very simple cases (addition but no multiplication required) • Solve two linear simultaneous equations in two variables in very simple cases (subtraction but no multiplication required) • Solve two linear simultaneous equations in two variables in very simple cases (addition or subtraction but no multiplication required) • Solve two linear simultaneous equations in two variables in simple cases (multiplication of one equation only required with addition) • Solve two linear simultaneous equations in two variables in simple cases (multiplication of one equation only required with subtraction) • Solve two linear simultaneous equations in two variables in simple cases (multiplication of one equation only required with addition or subtraction) • Derive and solve two simultaneous equations • Solve problems involving two simultaneous equations and interpret the solution

Prerequisites	Mathematical language	Pedagogical notes
• Solve linear equations • Substitute numbers into formulae • Plot graphs of functions of the form y = mx + c, x ± y = c and ax ± by = c) • Manipulate expressions by multiplying by a single term	Equation Simultaneous equation Variable Manipulate Eliminate Solve Derive Interpret	Students will be expected to solve simultaneous equations in more complex cases in Stage 10. This includes involving multiplications of both equations to enable elimination, cases where rearrangement is required first, and the method of substitution. NCETM: Glossary **Common approaches** *Students are taught to label the equations (1) and (2), and label the subsequent equation (3)* *Teachers use graphs (i.e. dynamic software) to demonstrate solutions to simultaneous equations at every opportunity*

Reasoning opportunities and probing questions	Suggested activities	Possible misconceptions
• Show me a solution to the equation 5a + b = 32. And another, and another … • Show me a pair of simultaneous equations with the solution x = 2 and y = -5. And another, and another … • Kenny and Jenny are solving the simultaneous equations x + 4y = 7 and x – 2y = 1. Kenny thinks the equations should be added. Jenny thinks they should be subtracted. Who do you agree with? Explain why.	KM: Stick on the Maths ALG2: Simultaneous linear equations NRICH: What's it worth? NRICH: Warmsnug Double Glazing NRICH: Arithmagons **Learning review** KM: 9M5 BAM Task	• Some students may think that addition of equations is required when both equations involve a subtraction • Some students may not multiply all coefficients, or the constant, when multiplying an equation • Some students may think that it is always right to eliminate the first variable • Some students may struggle to deal with negative numbers correctly when adding or subtracting the equations

Key concepts (GCSE subject content statements)

The Big Picture: Algebra progression map

- find approximate solutions to equations numerically using iteration
- solve two linear simultaneous equations in two variables algebraically

Return to overview

Possible themes

- Find approximate solutions to complex equations
- Solve simultaneous equations
- Solve problems involving simultaneous equations

Possible key learning points

- Understand the meaning of an iterative process
- Show that a solution to a complex equation lies between two given values
- Use an iterative formula to find approximate solutions to equations
- Use an iterative formula to find approximate solutions, to a given number of decimal places, to an equation
- Solve two linear simultaneous equations in two variables by substitution
- Solve two linear simultaneous equations in two variables by elimination (multiplication of both equations required)
- Solve two linear simultaneous equations in two variables by elimination (fractional coefficients)
- Derive and solve two simultaneous equations in complex cases
- Interpret the solution to a pair of simultaneous equations

Prerequisites

- Understand the concept of solving simultaneous equations by elimination
- Solve two linear simultaneous equations in two variables in very simple cases (no multiplication required)
- Solve two linear simultaneous equations in two variables in simple cases (multiplication of one equation only required)

Mathematical language

Unknown
Solve
Solution set
Interval
Decimal search
Iteration
Simultaneous equations
Substitution
Elimination

Notation
(a, b) for an open interval
[a, b] for a closed interval

Pedagogical notes

'Interval bisection' is often an intuitive approach used by pupils when faced with a certain type of problem (see below). 'Decimal search' includes 'trial and improvement' when the equation is not set to 0.
Having been introduced to iterative processes, iteration is explained as a process for finding approximate solutions to non-linear equations. GCSE examples can be found here.
Pupils have been introduced to solving simultaneous equations using elimination in simple cases in Stage 9. This includes either no multiplication being required or multiplication of just one equation being required. Solving simultaneous equations using substitution is new to this Stage.
NCETM: Departmental workshops: Simultaneous equations
NCETM: Glossary

Common approaches
Pupils are taught to label the equations (1) and (2), and label the subsequent equations (3), (4), etc.
Pupils are taught to use the 'ANS' key on their calculators when finding an approximate solution using iteration

Reasoning opportunities and probing questions

- Show me a pair of simultaneous equations with a solution x = 4, y = -2. And another. And another …
- Convince me x + 2y = 11, 3x + 4y = 18 can be solved using substitution and using elimination. Which method is best in this case?
- Always/ Sometimes/ Never: Solving a pair of simultaneous equations using elimination is more efficient than using substitution

Suggested activities

KM: Introduce iterative processes (in this example, interval bisection) by challenging students to find your chosen number (between 1 and 1000000) when the only clue is 'bigger' or smaller' after each guess. Compare the final number of guesses with 20 (since 2^{20} is close to 1000000 and students will probably have very quickly developed a process of roughly bisecting intervals).
KM: Babylonian square roots – an introduction to iterative processes
KM: Pre-iteration
KM: Iteration
KM: Stick on the Maths: ALG2 Simultaneous linear equations
KM: Convinced?: ALG2 Simultaneous linear equations
NRICH: Matchless
AQA: Bridging Units Resource Pocket 4 (Skills builder 2 and 3)
Learning review
GLOWMaths/JustMaths: Sample Questions Both Tiers
GLOWMaths/JustMaths: Sample Questions Higher Tiers
KM: 10M4 BAM Task

Possible misconceptions

- Some pupils may not check the solution to a pair of simultaneous equations satisfy both equations
- Some pupils may not multiply all coefficients, or the constant, when multiplying an equation
- Some pupils may struggle to deal with negative numbers correctly when adding or subtracting the equations

UNIT 9 – SOLVING QUADRATIC EQUATIONS GCSE EDEXCEL HIGHER TEXTBOOK

Key concepts (GCSE subject content statements)

The Big Picture: Algebra progression map

- solve quadratic equations by completing the square and by using the quadratic formula
- deduce turning points of quadratic functions by completing the square
- deduce roots of quadratic functions algebraically
- work with general iterative processes

Return to overview

Possible themes

- Solve quadratic equations
- Solve practical problems involving quadratic equations
- Understand and use iterative processes

Bring on the Maths: GCSE Higher Algebra

Solving quadratic equations: #6, #8, #9, #10, #11, #12

Possible key learning points

- Complete the square for a quadratic expression (a = 1)
- Complete the square for a quadratic expression (a > 1)
- Solve a quadratic equation (a = 1) by completing the square
- Solve a quadratic equation (a > 1) by completing the square
- Deduce the turning point of a quadratic function by completing the square
- Deduce the roots of a quadratic function using the completed square form
- Know and apply the formula for solving a simple quadratic equation of the form $ax^2 + bx + c = 0$
- Know and apply the formula for solving more complex quadratic equation of the form $ax^2 + bx + c = 0$
- Solve equations involving fractions that can be rearranged into the form $ax^2 + bx + c = 0$
- Solve problems in probability that generate a quadratic equation
- Solve problems involving quadratic equations
- Derive an iterative formula that can be used to find approximate solutions to a complex equation

Prerequisites

- Solve a quadratic equation by rearranging and factorising
- Identify when a quadratic equation cannot be solved by factorising
- Calculate fluently with negative numbers
- Rearrange algebraic expressions and equations
- Understand and use interval bisection
- Rearrange an equation to form an iterative formula

Mathematical language

(Quadratic) equation
Factorise
Rearrange
Complete the square
Unknown
Manipulate
Maximum, minimum
Parabola
Recurrence relation
Interval bisection

Notation

The form $(x + p)^2 – q$ usually implies that completing the square is required
Recurrence relations are equations such as $x_{n+1} = 2x_n - 3$

Pedagogical notes

Problems involving quadratic equations include, for example, shapes with dimensions expressed algebraically and area given.
Students have previously explored a range of iterative processes. They should now choose an appropriate method given the situation they are faced with.

Common approaches

All pupils investigate geometric representations of completing the square
All pupils derive the formula for solving a quadratic equation by completing the square

Reasoning opportunities and probing questions

- Always / sometimes / never: a quadratic equation has two solutions (justify using values of a, b and c)
- Show me an example of a quadratic equation with one solution. And another, and another, …
- Explore geometric representations of completing the square. Make connections between the geometry and the algebra to make sense of the name of the process

Suggested activities

NRICH: Proof sorter – quadratic equation
NRICH: Geometric parabola

Learning review

KM: 11M2 BAM Task, 11M3 BAM Task
GLOWMaths/JustMaths: Sample Questions Higher Tiers

Possible misconceptions

- Some students may attempt to always substitute positive values for a, b and c when using $ax^2 + bx + c = 0$
- Some students may forget that squaring a negative number results in a positive solution
- Some students may think that $(x + p)^2 – q$ implies that p must be positive

UNIT 10 – PROBABILITY | GCSE EDEXCEL HIGHER TEXTBOOK

Key concepts (GCSE subject content statements)

The Big Picture: Probability progression map

- calculate the probability of independent and dependent combined events, including using tree diagrams and other representations, and know the underlying assumptions
- enumerate sets and combinations of sets systematically, using tree diagrams
- understand that empirical unbiased samples tend towards theoretical probability distributions, with increasing sample size

Return to overview

Possible themes

- Understand and use tree diagrams
- Develop understanding of probability in situations involving combined events
- Use probability to make predictions

Possible key learning points

- List outcomes of combined events using a tree diagram
- Know and use the multiplication law of probability
- Now and use the addition law of probability
- Use a tree diagram to solve simple problems involving independent combined events
- Use a tree diagram to solve complex problems involving independent combined events
- Use a tree diagram to solve simple problems involving dependent combined events
- Use a tree diagram to solve complex problems involving dependent combined events
- Understand that relative frequency tends towards theoretical probability as sample size increases

Prerequisites

- Add fractions (decimals)
- Multiply fractions (decimals)
- Convert between fractions, decimals and percentages
- Use frequency trees to record outcomes of probability experiments
- Use experimental and theoretical probability to calculate expected outcomes

Mathematical language

Outcome, equally likely outcomes
Event, independent event, dependent event
Tree diagrams
Theoretical probability
Experimental probability
Random
Bias, unbiased, fair
Relative frequency
Enumerate
Set

Notation

P(A) for the probability of event A
Probabilities are expressed as fractions, decimals or percentage. They should not be expressed as ratios (which represent odds) or as words

Pedagogical notes

Tree diagrams can be introduced as simply an alternative way of listing all outcomes for multiple events. For example, if two coins are flipped, the possible outcomes can be listed (a) systematically, (b) in a two-way table, or (c) in a tree diagram. However, the tree diagram has the advantage that it can be extended to more than two events (e.g. three coins are flipped).
NCETM: Glossary

Common approaches

All students carry out the drawing pin experiment
Students are taught not to simply fractions when finding probabilities of combined events using a tree diagram (so that a simple check can be made that the probabilities sum to 1)

Reasoning opportunities and probing questions

- Show me an example of a probability problem that involves adding (multiplying) probabilities
- Convince me that there are eight different outcomes when three coins are flipped together
- Always / Sometimes / Never: increasing the number of times an experiment is carried out gives an estimated probability that is closer to the theoretical probability

Suggested activities

KM: Stick on the Maths: Tree diagrams
KM: Stick on the Maths: Relative frequency
KM: The drawing pin experiment

Learning review

KM: 9M13 BAM Task

Possible misconceptions

- When constructing a tree diagram for a given situation, some students may struggle to distinguish between how events, and outcomes of those events, are represented
- Some students may muddle the conditions for adding and multiplying probabilities
- Some students may add the denominators when adding fractions

Key concepts (GCSE subject content statements)

The Big Picture: Probability progression map

- apply systematic listing strategies including use of the product rule for counting
- calculate and interpret conditional probabilities through representation using expected frequencies with two-way tables, tree diagrams and Venn diagrams.

Return to overview

Possible themes

- Understand and use the product rule for counting
- Use Venn diagrams to represent probability situations
- Use two-way tables to represent probability situations
- Solve probability problems involving combined events

Possible key learning points

- Apply the product rule for counting
- Use a Venn diagram to sort information in a probability problem
- Use a two-way table to sort information in a probability problem
- Use a Venn diagram to calculate theoretical probabilities
- Use a two-way table to calculate theoretical probabilities
- Calculate conditional probabilities using different representations

Prerequisites

- Know when to add two or more probabilities
- Know when to multiply two or more probabilities
- Convert between fractions, decimals and percentages
- Use a tree diagram to calculate probabilities of dependent and independent combined events

Mathematical language

Outcome, equally likely outcomes
Event, independent event, dependent event
Tree diagrams
Theoretical probability, experimental probability
Random
Bias, unbiased, fair
Enumerate
Set
Conditional probability
Venn diagram

Notation

P(A) for the probability of event A
Probabilities are expressed as fractions, decimals or percentages. They should not be expressed as ratios (which represent odds) or as words

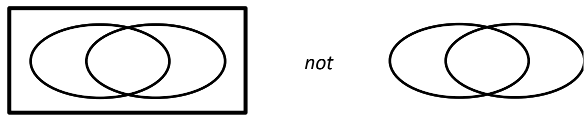
Pedagogical notes

In Stage 9, pupils calculate the probability of independent and dependent combined events using tree diagrams and enumerate sets and combinations of sets systematically, using tree diagrams. This unit has a strong emphasis on the use of Venn diagrams and two-way tables to solve probability problems.
Note: A Venn diagram has regions for all possible combinations of groups whether there are elements in those regions or not.
An Euler diagram only shows a region if things exist in that region.

NCETM: Glossary
NCETM: Department Workshops: Sets and Venn Diagrams
FMSP: Set Notation Poster

Common approaches

Pupils are taught to draw the border around the Venn 'regions' to highlight the elements that are not included in the regions.



Reasoning opportunities and probing questions

- Show me an example of a Venn diagram. And another. And another
- Show me an example of a two-way table. And another. And another
- Always / Sometimes / Never: All the regions of a Venn diagram must be populated

Suggested activities

CIMT: Venn Diagrams
OCR: Check In: Combined Events and Probability Diagrams
AQA: Bridging Unit: Set notation, number lines and Venn diagrams

Learning review

GLOWMaths/JustMaths: Sample Questions Both Tiers
GLOWMaths/JustMaths: Sample Questions Higher Tiers

Possible misconceptions

- When constructing a Venn diagrams for a given situation, some pupils may struggle to distinguish between elements that are included in the intersection of both regions or only in one of the regions
- Some pupils may muddle the conditions for adding and multiplying probabilities
- Some pupils may add the denominators when adding fractions

UNIT 11 – MULTIPLICATIVE REASONING	GCSE EDEXCEL HIGHER TEXTBOOK

Key concepts (GCSE subject content statements)

The Big Picture: Ratio and Proportion progression map

- solve problems involving direct and inverse proportion including graphical and algebraic representations
- apply the concepts of congruence and similarity, including the relationships between lengths in similar figures
- change freely between compound units (e.g. density, pressure) in numerical and algebraic contexts
- use compound units such as density and pressure

Return to overview

Possible themes

- Solve problems involving different types of proportion
- Investigate ways of representing proportion
- Understand and solve problems involving congruence
- Understand and solve problems involving similarity
- Know and use compound units in a range of situations

Possible key learning points

- Know the difference between direct and inverse proportion
- Recognise direct proportion in a situation
- Recognise inverse proportion in a situation
- Know the features of graphs that represent a direct or inverse proportion situation
- Know the features of expressions, or formulae, that represent a direct or inverse proportion situation
- Understand the connection between the multiplier, the expression and the graph
- Solve problems involving direct proportion
- Solve problems involving inverse proportion
- Identify congruence of shapes in a range of situations
- Finding missing lengths in similar shapes
- Convert between compound units of density and pressure
- Solve problems involving density
- Solve problems involving pressure
- Solve more complex problems involving speed

Prerequisites

- Find a relevant multiplier in a situation involving proportion
- Plot the graph of a linear function
- Understand the meaning of a compound unit
- Convert between units of length, capacity, mass and time

Mathematical language

Direct proportion
Inverse proportion
Multiplier
Linear
Congruent, Congruence
Similar, Similarity
Compound unit
Density, Population density
Pressure

Notation
Kilograms per metre cubed is written as kg/m^3

Pedagogical notes

Students have explored enlargement previously.
Conditions for congruent triangles will be explored in a future unit.
Use the story of Archimedes and his 'eureka moment' when introducing density.
Up-to-date information about population densities of counties and cities of the UK, and countries of the world, is easily found online.
NCETM: The Bar Model
NCETM: Multiplicative reasoning
NCETM: Departmental workshops: Proportional Reasoning
NCETM: Departmental workshops: Congruence and Similarity
NCETM: Glossary
Common approaches
All students are taught to set up a 'proportion table' and use it to find the multiplier in situations involving direct proportion

Reasoning opportunities and probing questions

- Show me an example of two quantities that will be in direct (inverse) proportion. And another. And another …
- Convince me that this information shows a proportional relationship. What type of proportion is it?

40	3
60	2
80	1.5

- Which is the greatest density: 0.65g/cm^3 or 650kg/m^3? Convince me.

Suggested activities

KM: Graphing proportion
NRICH: In proportion
NRICH: Ratios and dilutions
NRICH: Similar rectangles
NRICH: Fit for photocopying
NRICH: Tennis
NRICH: How big?
Learning review
KM: 9M7 BAM Task

Possible misconceptions

- Many students will want to identify an additive relationship between two quantities that are in proportion and apply this to solve problems
- The word 'similar' means something much more precise in this context than in other contexts students encounter. This can cause confusion.
- Some students may think that a multiplier always has to be greater than 1

UNIT 11 – MULTIPLICATIVE REASONING   GCSE EDEXCEL HIGHER TEXTBOOK

Key concepts (GCSE subject content statements)

The Big Picture: Ratio and Proportion progression map

- interpret equations that describe direct and inverse proportion
- recognise and interpret graphs that illustrate direct and inverse proportion
- understand that X is inversely proportional to Y is equivalent to X is proportional to 1/Y

Return to overview

Possible themes

- Explore differences between direct and inverse proportion
- Investigate ways of representing proportion in situation
- Solve problems involving proportion

Possible key learning points

- Recognise and interpret graphs that illustrate direct proportion
- Recognise and interpret graphs that illustrate inverse proportion
- Understand that X is inversely proportional to Y is equivalent to X is proportional to 1/Y
- Interpret equations that describe direct proportion
- Interpret equations that describe inverse proportion
- Solve problems which include finding the multiplier in a situation involving direct proportion
- Solve problems which include finding the multiplier in a situation involving inverse proportion

Prerequisites

- Know the difference between direct and inverse proportion
- Recognise direct or inverse proportion in a situation
- Know the features of a graph that represents a direct or inverse proportion situation
- Know the features of an expression (or formula) that represents a direct or inverse proportion situation
- Understand the connection between the multiplier, the expression and the graph

Mathematical language

Direct proportion
Inverse proportion
Multiplier

Notation
$\propto$ - 'proportional to'

Pedagogical notes

Pupils have solved simple problems involving direct and inverse proportion in Stage 9. This unit focuses on developing a formal algebraic approach, including the use of proportionality constants, to solve direct and inverse proportion problems.

NCETM: Departmental workshops: Proportional Reasoning
NCETM: Glossary

Common approaches
All pupils are taught to find a proportionality constant when solving problems

Reasoning opportunities and probing questions

- Show me an example of two quantities that will be in direct proportion. And another. And another …
- Convince me that this information shows a proportional relationship. What type of proportion is it?

40	50
60	75
80	100

- Always/Sometimes/Never: X is inversely proportional to Y is equivalent to X is proportional to 1/Y

Suggested activities

KM: Graphing proportion
KM: Investigating proportionality 2
KM: Stick on the Maths NNS1: Understanding Proportionality
KM: Stick on the Maths CALC1: Proportional Change and multiplicative methods
KM: Convinced: NNS1: Understanding Proportionality
KM: Convinced: CALC1: Proportional Change and multiplicative methods
Hwb: Inverse or direct?
NRICH: In Proportion

Learning review
GLOWMaths/JustMaths: Sample Questions Both Tiers
GLOWMaths/JustMaths: Sample Questions Higher Tiers
KM: 10M2 BAM Task

Possible misconceptions

- Some pupils will want to identify an additive relationship between two quantities that are in proportion and apply this to solve problems
- Some pupils may interpret *'x is inversely proportional to y'* as y=x/k rather than y = k/x
- Some pupils may think that the proportionality constant always has to be greater than 1

Key concepts (GCSE subject content statements)

The Big Picture: Properties of Shape progression map

- use the basic congruence criteria for triangles (SSS, SAS, ASA, RHS)
- apply angle facts, triangle congruence, similarity and properties of quadrilaterals to conjecture and derive results about angles and sides, including Pythagoras' Theorem and the fact that the base angles of an isosceles triangle are equal, and use known results to obtain simple proofs

Return to overview

Possible themes

- Explore the congruence of triangles
- Investigate geometrical situations
- Form conjectures
- Create a mathematical proof

Possible key learning points

- Apply angle facts to derive results about angles and sides
- Create a geometrical proof
- Know the conditions for triangles to be congruent
- Use the conditions for congruent triangles
- Use congruence in geometrical proofs
- Solve geometrical problems involving similarity
- Know the meaning of a Pythagorean triple

Prerequisites

- Know angle facts including angles at a point, on a line and in a triangle
- Know angle facts involving parallel lines and vertically opposite angles
- Know the properties of special quadrilaterals
- Know Pythagoras' theorem

Mathematical language

Congruent, congruence
Similar (shapes), similarity
Hypotenuse
Conjecture
Derive
Prove, proof
Counterexample

Notation
Notation for equal lengths and parallel lines
SSS, SAS, ASA, RHS
The 'implies that' symbol ($\Rightarrow$)

Pedagogical notes

'Known facts' should include angle facts, triangle congruence, similarity and properties of quadrilaterals
NCETM: Glossary

Common approaches
All students are asked to draw 1, 2, 3 and 4 points on the circumference of a set of circles. In each case, they join each point to every other point and count the number of regions the circle has been divided into. Using the results 1, 2, 4 and 8 they form a conjecture that the sequence is the powers of 2. They test this conjecture for the case of 5 points and find the circle is divided into 16 regions as expected. Is this enough to be convinced? It turns out that it should not be, as 6 points yields either 30 or 31 regions depending on how the points are arranged. This example is used to emphasise the importance and power of mathematical proof. See KM: Geometrical proof

Reasoning opportunities and probing questions

- Show me a pair of congruent triangles. And another. And another
- Show me a pair of similar triangles. And another. And another
- What is the same and what is different: Proof, Conjecture, Justification, Test?
- Convince me the base angles of an isosceles triangle are equal.
- Show me a Pythagorean Triple. And another. And another.
- Convince me a triangle with sides 3, 4, 5 is right-angled but a triangle with sides 4, 5, 6 is not right-angled.

Suggested activities

KM: Geometrical proof
KM: Don't be an ASS
KM: Congruent triangles
KM: Unjumbling and examining angles
KM: Shape work: Triangles to thirds, 4×4 square, Squares, Congruent triangles
KM: Triple triplicate and Pythagorean triples
KM: Daniel Gumb's cave
NRICH: Tilted squares
NRICH: What's possible?

Bring on the Maths
Level 8: Congruence and similarity
Year 9, Logic: Triangles, More triangles

Learning review
KM: 9M12 BAM Task, 9M9 BAM Task
KM: Quiz and Review

Possible misconceptions

- Some students think AAA is a valid criterion for congruent triangles.
- Some students try and prove a geometrical situation using facts that 'look OK', for example, *'angle ABC looks like a right angle'*.
- Some students do not appreciate that diagrams are often drawn to scale.
- Some students think that all triangles with sides that are consecutive numbers are right angled.

UNIT 12 – SIMILARITY & CONGRUENCE GCSE EDEXCEL HIGHER TEXTBOOK

Key concepts (GCSE subject content statements)

The Big Picture: Measurement and mensuration progression map

- calculate surface area and volume of spheres, pyramids, cones and composite solids
- apply the concepts of congruence and similarity, including the relationships between length, areas and volumes in similar figures

Return to overview

Possible themes

- Calculate surface areas of solids
- Calculate volumes of solids
- Solve problems involving enlargement and 3D shapes

Possible key learning points

- Use Pythagoras' theorem to find lengths in a pyramid or cone
- Find the surface area of spheres, cones and pyramids
- Find the volume of spheres, cones and pyramids
- Identify how to find the volume or surface area of a composite solid
- Solve practical problems involving the surface area of solids
- Solve practical problems involving the volume of solids
- Understand the implications of enlargement on area
- Understand the implications of enlargement on volume
- Move freely between scale factors for length, area and volume
- Solve practical problems involving length, area and volume in similar figures

Prerequisites

- Calculate exactly with multiples of π
- Know and use the formula for area and circumference of a circle
- Know how to use formulae to find the area of rectangles, parallelograms, triangles, trapezia, circles, sectors and
- Know how to find the area of compound shapes
- Know how to find the surface area of a right prism and a cylinder
- Calculate the surface area of a right prism and a cylinder
- Carry out an enlargement
- Find the scale factor of a given enlargement
- Use Pythagoras' theorem to find missing lengths in right-angled triangles

Mathematical language

(Composite) solid
Sphere, Pyramid, Cone
Perpendicular (height), (slant height)
Surface area
Volume
Congruent, congruence
Similarity, similar shapes, similar figures
Enlarge, enlargement
Scale factor

Notation

π
Abbreviations of units in the metric system: km, m, cm, mm, mm^2, cm^2, m^2, km^2, mm^3, cm^3, km^3

Pedagogical notes

Pupils have previously learnt how to find the surface area of right prisms and cylinders in Stage 9. This unit focuses on finding the volume and surface areas of cones, spheres and pyramids.
Pupils also explore congruence and similarity - the use of proportion tables can be helpful to find the multiplier when solving similarity problems such as:

	Shape A	Shape B
Known lengths	6	9
Missing lengths	10	15

→ × 1.5 →

NCETM: Glossary

Common approaches

Pupils explore the surface area of spheres using oranges (https://www.youtube.com/watch?v=cAxHYFRx1Fs)
Pupils explore volumes of pyramids by making nets of pyramids and prisms with the same polygonal base and using sand or sugar to compare volumes.

Reasoning opportunities and probing questions

- Always/ Sometimes/ Never: The value of the volume of a pyramid is less than the value of the surface area of a pyramid.
- Always/ Sometimes/ Never: The value of the volume of a sphere is less than the value of the surface area of a sphere.
- Convince me that the volume of a pyramid = $^1/_3$ × A × h
- Convince me that 1 m^3 = 1 000 000 cm^3

Suggested activities

KM: Stick on the Maths 8: Congruence and Similarity
KM: Convinced? Congruence and Similarity
NRICH: Surface Area and Volume and Nicely Similar
Hwb: Summerhouse and Radiators
OCR: Congruence Check In and Similarity Check In

Learning review

GLOWMaths/JustMaths: Sample Questions Both Tiers
GLOWMaths/JustMaths: Sample Questions Higher Tiers
KM: 10M11 BAM Task

Possible misconceptions

- Some pupils will work out 4/3 × $(\pi \times r)^3$ when finding the volume of a sphere.
- Some pupils may confuse the concepts of surface area and volume
- Some pupils will work out 4 × $(\pi \times r)^2$ when finding the surface area of a sphere.
- Some pupils may think the volume of a pyramid = ½ × A × h

Key concepts (GCSE subject content statements)

The Big Picture: Properties of Shape progression map

- know the formulae for Pythagoras' theorem, $a^2 + b^2 = c^2$, and apply it to find lengths in three dimensional figures
- know the trigonometric ratios, sinθ = opposite/hypotenuse, cosθ = adjacent/hypotenuse, tanθ = opposite/adjacent and apply them to find angles and lengths in three dimensional figures
- know and apply the sine rule, a/sinA = b/sinB = c/sinC, and the cosine rule, $a^2 = b^2 + c^2 - 2bc \cos A$, to find unknown lengths and angles
- know and apply area = ½ab sinC to calculate the area, sides or angles of any triangle

Return to overview

Possible themes

- Explore three-dimensional shapes
- Apply Pythagoras' theorem in three dimensions
- Apply trigonometry in three dimensions
- Know and use the sine rule
- Know and use the cosine rule

Bring on the Maths: GCSE Higher Shape
Investigating triangles: #6
Investigating angles: #10, #13

Possible key learning points

- Use Pythagoras' theorem to find the length of a given diagonal in a cuboid
- Use Pythagoras' theorem to find any length in a cuboid
- Use Pythagoras' theorem to find missing lengths in other three dimensional figures
- Use Pythagoras' theorem to solve problems involving three dimensional figures
- Use trigonometry to find the angle between a line and a plane
- Solve simple problems involving missing lengths and angles in three dimensional figures
- Solve more complex problems involving missing lengths and angles in three dimensional figures
- Know and use the sine rule in simple cases
- Use the sine rule to find a missing side in a non-right angled triangle
- Use the sine rule to find a missing angle(s) in a non-right angled triangle
- Know and use the cosine rule in simple cases
- Use the cosine rule to find a missing side in a non-right angled triangle
- Use the cosine rule to find a missing angle in a non-right angled triangle
- Solve complex problems involving bearings
- Know and use area = ½ab sinC to calculate the area of any triangle
- Know and use area = ½ab sinC to calculate sides or angles of any triangle

Prerequisites

- Apply Pythagoras' theorem in two dimensions
- Know the trigonometric ratios, sinθ = opp/hyp, cosθ = adj/hyp, tanθ = opp/adj
- Choose an appropriate trigonometric ratio that can be used in a given two-dimensional situation
- Set up and solve a trigonometric equation to find a missing side or angle in a right-angled triangle

Mathematical language

Diagonal (Face Diagonal, Space Diagonal)
Plane
Opposite, Adjacent, Hypotenuse
Trigonometry
Sine, Cosine, Tangent
Angle of elevation, angle of depression

Notation
sinθ stands for the 'sine of θ'
$\sin^{-1}$ is the inverse sine function, and not 1 ÷ sin

Pedagogical notes

Ensure that all students are aware of the importance of their scientific calculator being in degrees mode.
Ensure that students do not round until the end of a multi-step calculation
This unit of trigonometry should focus on right-angled triangles in three dimensions and non-right-angled triangles.
NRICH: History of Trigonometry
NCETM: Glossary

Common approaches
All students explore how to derive the sine rule

Reasoning opportunities and probing questions

- Show me a diagonal/right angle in this cuboid. And another, and another …
- Convince me that you have chosen the correct trigonometric fact
- What's the same and what is different: $a^2 = b^2 + c^2 - 2bc \cos A$ and $a^2 = b^2 + c^2$. Can you find a connection?
- What's the same and what is different: area = ½ab sinC and area = ½bh. Can you find a connection?

Suggested activities

KM: Investigate Euler bricks
Hwb: Q8 Triangle Side Length
Hwb: T3 Greenhouse
Hwb: T20 Wardrobe
NRICH: Raising the Roof
NRICH: Coke Machine
NRICH: Cosines Rule

Learning review
KM: 11M7 BAM Task
GLOWMaths/JustMaths: Sample Questions Higher Tiers

Possible misconceptions

- Some students may label opposite and adjacent in a non-right-angled triangle
- Some students may not balance an equation such as 5 = 4/sinθ correctly, believing that the next step is sinθ = 5/4
- Some students may think that $\cos^{-1}θ$ = 1 ÷ cosθ

UNIT 14 – STATISITICS & SAMPLING	GCSE EDEXCEL HIGHER TEXTBOOK

Key concepts (GCSE subject content statements)

The Big Picture: Statistics progression map

- infer properties of populations or distributions from a sample, whilst knowing the limitations of sampling
- construct and interpret diagrams for grouped discrete data and continuous data, i.e. cumulative frequency graphs, and know their appropriate use
- interpret, analyse and compare the distributions of data sets from univariate empirical distributions through appropriate graphical representation involving discrete, continuous and grouped data, including box plots
- interpret, analyse and compare the distributions of data sets from univariate empirical distributions through appropriate measures of central tendency including quartiles and inter-quartile range

Return to overview

Possible themes	Possible key learning points
• Construct and interpret cumulative frequency graphs • Construct and interpret box plots • Analyse distributions of data sets	• Use a sample to infer properties of a population • Understand the limitations of sampling • Know the meaning of the lower quartile and upper quartile • Find the quartiles for discrete data sets • Calculate and interpret the interquartile range • Construct and interpret a box plot for discrete data • Use box plots to compare distributions • Understand the meaning of cumulative frequency • Complete a cumulative frequency table • Construct a cumulative frequency curve • Use a cumulative frequency curve to estimate the quartiles for grouped continuous data sets • Use a cumulative frequency curve to estimate properties of grouped continuous data sets

Prerequisites	Mathematical language	Pedagogical notes
• Know the meaning of discrete and continuous data • Interpret and construct frequency tables • Analyse data using measures of central tendency	Categorical data, Discrete data Continuous data, Grouped data Axis, axes Population Sample Cumulative frequency Box plot, box-and-whisker diagram Central tendency Mean, median, mode Spread, dispersion, consistency Range, Interquartile range Skewness **Notation** Correct use of inequality symbols when labeling groups in a frequency table	In Stage 8, pupils explore how to find the modal class of set of grouped data, the class containing the median of a set of data, the midpoint of a class, an estimate of the mean from a grouped frequency table and an estimate of the range from a grouped frequency table This unit builds on the knowledge by exploring measures of central tendency using quartiles and inter-quartile range. Cumulative frequency curves are usually S-shaped, known as an ogive. Box plots are also known as 'box and whisker' plots. NCETM: Glossary **Common approaches** *The median is calculated by finding the (n+1)/2 th item and the lower quartile by finding the (n+1)/4 th item unless n is large (n>30). In the case when n>30, n/2 and n/4 can be used to find the median and lower quartile.*

Reasoning opportunities and probing questions	Suggested activities	Possible misconceptions

• Show me a box plot with a large/small interquartile range. And another. And another. • What's the same and what's different: inter-quartile range, median, mean, mode • Convince me how to construct a cumulative frequency curve • Always/Sometimes/Never: The median is greater than the inter-quartile range	KM: Stick on the Maths HD1: Statistics, HD2: Comparing Distributions KM: Cumulative Frequency and Box Plots NRICH: The Live of Presidents NRICH: Olympic Triathlon NRICH: Box Plot Match OCR: Sampling, Analysing Data **Learning review** GLOWMaths/JustMaths: Sample Questions Both Tiers GLOWMaths/JustMaths: Sample Questions Higher Tiers KM: 10M13 BAM Task	• Some pupils may plot the cumulative frequencies against the midpoints or lower bounds of grouped data • Some pupils may try to construct a cumulative frequency curve by plotting the frequencies against the upper bound of grouped data • Some pupils may try to construct a cumulative frequency curve by joining the points with straight lines rather than a smooth curve • Some pupils may forget to add the 'whiskers' when constructing a 'box and whisker' plot.

UNIT 14 – STATISTICS & SAMPLING — *GCSE EDEXCEL HIGHER TEXTBOOK*

Key concepts (GCSE subject content statements)

The Big Picture: Statistics progression map

- construct and interpret diagrams for grouped discrete data and continuous data, i.e. histograms with equal and unequal class intervals and know their appropriate use

Return to overview

Possible themes	Possible key learning points
• Construct and interpret histograms • Analyse distributions of data sets • Solve problems involving histograms **Bring on the Maths: GCSE Higher Data** Representing Data: #7 Interpreting and Discussing: #9	• Understand the definition of a histogram • Construct histograms for grouped data with unequal class intervals • Use a histogram to find missing values in a frequency table • Use a partially completed histogram and frequency table to complete both • Solve problems involving histograms

Prerequisites	Mathematical language	Pedagogical notes
• Know the meaning of continuous data • Understand and use grouped frequency tables • Interpret histograms for grouped data with equal class intervals	Continuous data, Grouped data Table, Frequency table Frequency Frequency density Histogram Scale, Graph Axis, axes **Notation** Correct use of inequality symbols when labeling groups in a frequency table	The word histogram is often misused and an internet search of the word will usually reveal a majority of non-histograms. The correct definition is 'a diagram made of rectangles whose areas are proportional to the frequency of the group'. If the class widths are equal, then the vertical axis shows the frequency. It is only later that pupils need to be introduced to unequal class widths and frequency density. NCETM: Glossary **Common approaches**
Reasoning opportunities and probing questions	**Suggested activities**	**Possible misconceptions**
• Convince Kenny how to construct a histogram with unequal intervals • What's the same and what is different: histogram, bar chart? • Always/Sometimes/Never: The value of the frequency density is less than 1 • Kenny thinks that histogram is just a 'fancy' name for a bar chart. Do you agree with Kenny? Explain your answer.	KM: What the heck is a histogram? KM: Stick on the Maths HD3: Working with grouped data AQA Maths: Collecting and representing data **Learning review** GLOWMaths/JustMaths: Sample Questions Higher Tiers	• Some pupils may think that histogram is a 'posh term' for a bar chart • Some pupils may calculate the frequency density incorrectly such as dividing the bar width by the frequency. • Some pupils may label the bar of a histogram rather than the boundaries of the bars • Some pupils may leave gaps between the bars in a histogram • Some pupils may misuse the inequality symbols when working with a grouped frequency table

UNIT 15 – QUADRATICS

Key concepts (GCSE subject content statements)

The Big Picture: Algebra progression map

- simplify and manipulate algebraic expressions involving algebraic fractions
- manipulate algebraic expressions by expanding products of more than two binomials
- simplify and manipulate algebraic expressions (including those involving surds) by expanding products of two binomials and factorising quadratic expressions of the form $x^2 + bx + c$, including the difference of two squares
- manipulate algebraic expressions by factorising quadratic expressions of the form $ax^2 + bx + c$

Return to overview

Possible themes

- Manipulate algebraic fractions
- Manipulate algebraic expressions

Possible key learning points

- Add and subtract algebraic fractions
- Multiply and divide algebraic fractions
- Simplify an algebraic fraction
- Expand the product of three binomials
- Expand the product of two binomials involving surds
- Factorise an expression involving the difference of two squares
- Factorise a quadratic expression of the form $ax^2 + bx + c$ (a is prime)
- Factorise a quadratic expression of the form $ax^2 + bx + c$ (a is composite)
- Identify when factorisation of the numerator and/or denominator is needed to simplify an algebraic fraction
- Simplify an algebraic fraction that involves factorisation
- Change the subject of a formula when more than two steps are required
- Change the subject of a formula when the required subject appears twice

Prerequisites

- Calculate with negative numbers
- Multiply two linear expressions of the form $(x \pm a)(x \pm b)$
- Factorise a quadratic expression of the form $x^2 + bx + c$
- Add, subtract, multiply and divide proper fractions
- Change the subject of a formula when two steps are required

Mathematical language

Equivalent
Equation
Expression
Expand
Linear
Quadratic
Algebraic Fraction
Difference of two squares
Binomial
Factorise

Notation

Pedagogical notes

Pupils have applied the four operations to proper, and improper, fractions in Stage 7 and factorised quadratics of the form $x^2 + bx + c$ in Stage 9. Pupils should build on the experiences of using the grid method in Stage 9 to expand products of more than two binomials.

Eg $(x + 2)(x + 3)(x – 4) = (x^2 + 5x + 6)(x – 4) = x^3 + x^2 – 14x – 24$

	x^2	$+5x$	$+6$
x	x^3	$+5x^2$	$+6x$
$–4$	$–4x^2$	$–20x$	$–24$

Teachers also need to help pupils 'see' the difference of two squares by using pictorial representations

NCETM: Algebra
NCETM: Glossary

Common approaches

Students are taught to use the grid method in reverse to factorise a quadratic
Students manipulate algebra tiles to explore factoring quadratics
The difference of two squares is explained using visual representation

Reasoning opportunities and probing questions

- The answer is $2x^2 + 10x + c$. Show me a possible question. And another.
- Kenny simplifies $\frac{3x^2+x}{x}$ as $3x^2 + 1$. Do you agree with Kenny? Explain.
- Convince me that $103^2 – 97^2 = 1200$ without a calculator.
- Convince me that $4x^2 – 9 \equiv (3x – 2)(3x + 2)$.
- Jenny thinks that $(3x – 2)^2 = 3x^2 + 12x + 4$. Do you agree with Jenny? Explain your answer.
- Convince me that $\frac{2x^2+5x+2}{2x+1} = x + 2$

Suggested activities

KM: Simplifying algebraic fractions
KM: Maths to Infinity: Brackets and Quadratics
KM: Stick on the Maths: Quadratic sequences
NRICH: What's possible?
NRICH: Finding Factors
Algebra Tiles (external site)
Learning review
GLOWMaths/JustMaths: Sample Questions Higher Tiers
KM: 10M5 BAM Task

Possible misconceptions

- Once pupils know how to factorise a quadratic expression of the form $x^2 + bx + c$ they might overcomplicate the simpler case of factorising an expression such as $3x^2 + 6x$ ($\equiv (3x + 0)(x + 2)$)
- Some pupils may think that $(x + a)^2 \equiv x^2 + a^2$
- Some pupils may apply the 'rules of factorising' quadratics of the form $x^2 + bx + c$ to quadratics of the form $ax^2 + bx + c$; e.g. $2x^2 + 7x + 10 \equiv (2x + 5)(x + 2)$ because $2 \times 5 = 10$ and $2 + 5 = 7$.

UNIT 15 – QUADRATICS	GCSE EDEXCEL HIGHER TEXTBOOK

Key concepts (GCSE subject content statements)

The Big Picture: Algebra progression map

- solve quadratic equations algebraically by factorising
- solve quadratic equations (including those that require rearrangement) algebraically by factorising
- find approximate solutions to quadratic equations using a graph
- deduce roots of quadratic functions algebraically

Return to overview

Possible themes

- Solve quadratic equations
- Use graphs to solve equations

Possible key learning points

- Solve a quadratic equation of the form $x^2 + bx + c = 0$ by factorising
- Solve a quadratic equation by rearranging and factorising
- Make connections between graphs and quadratic equations of the form $ax^2 + bx + c = 0$
- Make connections between graphs and quadratic equations of the form $ax^2 + bx + c = dx + e$
- Find approximate solutions to quadratic equations using a graph
- Deduce roots of quadratic functions algebraically
- Solve problems that involve solving a quadratic equation in context

Prerequisites

- Manipulate linear equations
- Factorise a quadratic expression of the form $x^2 + bx + c$
- Factorise a quadratic expression of the form $ax^2 + bx + c$
- Make connections between a linear equation and a graph

Mathematical language

(Quadratic) equation
Factorise
Rearrange
Variable
Unknown
Manipulate
Solve
Deduce
x-intercept
Root

Pedagogical notes

Pupils factorise quadratic expressions of the form $ax^2 + bx + c$ in Stage 9 ($a = 1$) and Stage 10.
If $A \times B = 0$ then either $A = 0$ or $B = 0$ is a fundamental underlying concept to solving quadratic equations when $b \neq 0$ and $c \neq 0$ by factorising.
Pupils should experience solving quadratics with $b \neq 0$ and $c = 0$, such as $x^2 + 6x = 0$, and quadratics with $b \neq 0$ and $c \neq 0$, such as $x^2 + 6x + 8 = 0$.
Pupils may wish to 'divide both sides by 'x' when solving quadratics such as $x^2 + 6x = 0$ without appreciating that x could equal zero.
NCETM: Glossary

Common approaches

Pupils are taught how to solve quadratics of the form $ax^2 + bx + c = 0$ when:
- $b = 0$, $b \neq 0$ and $c = 0$, $b \neq 0$ and $c \neq 0$

Pupils are encouraged, whenever possible, to divide a quadratic equation by a common factor to make the factorising process easier, such as $2x^2 + 6x + 8 = 0$

Reasoning opportunities and probing questions

- Show me a quadratic equation that can be solved by factorising. And another, and another …
- Show me a quadratic equation with one solution $x = 3$. And another, and another …
- Always/Sometimes/Never: A quadratic equation can be solved by factorising.
- Convince me why you can't '*divide both sides by x*' when solving $x^2 + 8x = 0$
- Kenny is solving $x^2 + 6x + 8 = 2$ as follows:
  $(x + 4)(x + 2) = 2$ so $x + 4 = 2$ or $x + 2 = 1$.
  Therefore, $x = -2$ and $x = -1$.
- Do you agree with Kenny? Explain your answer.

Suggested activities

NRICH: How old am I?
NRICH: Golden thoughts
Hwb: Algebra Fails

Learning review

GLOWMaths/JustMaths: Sample Questions Both Tiers
GLOWMaths/JustMaths: Sample Questions Higher Tiers
KM: 10M6 BAM Task, 10M7 BAM Task

Possible misconceptions

- Some pupils may not appreciate that a quadratic equation must equal zero when solving by factorising
- Some pupils may solve $x^2 + 8x = 0$ by dividing both sides by x to get $x + 8 = 0$, $x = -8$.
- Some pupils may forget to divide by the coefficient of x when solving quadratics such as $2x^2 + 5x + 2 = 0$, i.e. $(2x + 1)(x + 2) = 0$ so $2x + 1 = 0$ or $x + 2 = 0$ and therefore $x = -1$ (rather than $-½$ or $x = -2$)
- Some pupils may not divide a quadratic equation by a common factor to make the factorising process easier, such as $2x^2 + 6x + 8 = 0$

UNIT 15 – QUADRATICS & CIRCLES	GCSE EDEXCEL HIGHER TEXTBOOK	6 lessons

Key concepts (GCSE subject content statements)

The Big Picture: Algebra progression map

- use the form $y = mx + c$ to identify perpendicular lines
- recognise and use the equation of a circle with centre at the origin
- find the equation of a tangent to a circle at a given point

Return to overview

Possible themes	Possible key learning points
• Investigate features of straight line graphs • Know and use the equation of a circle with centre at the origin • Solve problems involving the equation of a circle	• Know that perpendicular lines have gradients with a product of -1 • Identify perpendicular lines using algebraic methods • Identify the equation of a circle from its graph • Use the equation of a circle to draw its graph • Find the equation of a tangent to circle at a given point • Solve algebraic problems involving tangents to a circle

Prerequisites	Mathematical language	Pedagogical notes
• Use the form y = mx + c to identify parallel lines • Rearrange an equation into the form y = mx + c • Find the equation of a line through one point with a given gradient • Find the equation of a line through two given points • Know and apply Pythagoras' Theorem	Function, equation Linear, non-linear Parallel Perpendicular Gradient y-intercept, x-intercept, root Sketch, plot Centre (of a circle) Radius Tangent **Notation** $y = mx + c$	This unit builds on the graphs of linear functions from Stage 9 including parallel lines. Exploring the equation of circle is new for the pupils and it is important to check students know the definition of a circle (i.e. the locus of points from a fixed point) to help understand how to derive the general formula $(x – a)^2 + (y – b)^2 = r^2$ by applying Pythagoras' theorem to find the distance of (x,y), a general point on the circumference of the circle, from (a,b), the centre of a circle with radius r. NCETM: Glossary **Common approaches** *All student use dynamic graphing software to explore perpendicular graphs – i.e. plot two perpendicular lines and analyse the relationship between the gradients of the two lines.* *Pupils plot points with a 'x' and not '•'* *Pupils draw graphs in pencil*

Reasoning opportunities and probing questions	Suggested activities	Possible misconceptions
• Show me the equation of two lines that are perpendicular to each other. And another. And another. • Convince me the lines y + 0.5x = 7, 6 – x = 2y and 8 + 2y + 4x = 0 are perpendicular to y = 3 + 2x. • Show me the equation of a circle - what is the centre and radius of the circle? And another. And another. • True or False? A straight line can intersect a circle at 0, 1 or 2 points. • Convince me how to find the equation of a tangent to a circle at a given point	KM: The gradient of perpendicular lines KM: Introducing the equation of a circle KM: The general equation of a circle KM: The general equation of a circle NRICH: Perpendicular lines NRICH: At Right Angles FMSP: Geogebra – Equation of a tangent to a circle **Learning review** GLOWMaths/JustMaths: Sample Questions Both Tiers GLOWMaths/JustMaths: Sample Questions Higher Tiers KM: 10M9 BAM Task	• Some pupils do not rearrange the equation of a straight line correctly to find the gradient of a straight line. For example, they think that the line y – 2x = 6 has a gradient of -2. • Some pupils may think that gradient = (change in x) / (change in y) when trying to equation of a line through two given points. • Some pupils may think that the equation of a circle is $(x–a)^2+(y–b)^2 = r$

UNIT 16 – CIRCLE THEOREMS | GCSE EDEXCEL HIGHER TEXTBOOK

Key concepts (GCSE subject content statements)

The Big Picture: [Properties of Shape progression map]

- apply and prove the standard circle theorems concerning angles, radii, tangents and chords, and use them to prove related results

Return to overview

Possible themes

- Investigate geometric patterns using circles
- Explore circle theorems
- Make and prove conjectures

Bring on the Maths: GCSE Higher Shape
Investigating angles in circles: #1, #2, #3, #4

Possible key learning points

- Create a chain of logical steps to create a proof in a geometrical situation
- Know that 'the angle in a semicircle is a right angle'
- Know that 'the angle at the centre is double the angle at the circumference'
- Know that 'angles in the same segment are equal'
- Know that 'opposite angles in a cyclic quadrilateral sum to 180°'
- Know that 'two tangents from an external point are equal in length'
- Know that 'a radius is perpendicular to a tangent at that point'
- Know that 'a radius that bisects a chord is perpendicular to that chord'
- Know the alternate segment theorem
- Use a combination of known and derived facts to solve a geometrical problem
- Identify when a circle theorem can be used to help solve a geometrical problem
- Justify solutions to geometrical problems

Prerequisites

- Know the vocabulary of circles
- Know angle facts including angles at a point, on a line and in a triangle
- Know angle facts involving parallel lines and vertically opposite angles
- Know the properties of special quadrilaterals

Mathematical language

Radius, radii
Tangent
Chord
Theorem
Conjecture
Derive
Prove, proof
Counterexample

Notation
Notation for equal lengths and parallel lines
The 'implies that' symbol (⇒)

Pedagogical notes

Students should also explore the following (paraphrased) circle theorems:

- Cyclic Quadrilateral: GSP, Word
- Radius and Tangent: GSP, Word
- Radius and chord:
- Angles in the Same Segment: GSP, Word
- The Angle in the Centre: GSP, Word
- Two Tangents: GSP, Word
- Alternate Segment Theorem: GSP, Word

NCETM: Glossary

Common approaches
All students are first introduced to the idea of circle theorems by investigating Thales Theorem. This is then extended to demonstrate that 'the angle at the centre is twice the angle at the circumference'
All students are given the opportunity to create and explore dynamic diagrams of different circle theorems.

Reasoning opportunities and probing questions

- How can you use a set square to find the centre of a circle?
- Show me a radius of this circle. And another, and another … (What does this tell you about the lengths? About the triangle?)
- Provide the steps for a geometrical proof of a circle theorem and ask students to 'unjumble' them and create the proof, explaining their thinking at each step
- Use the 'Always / Sometimes / Never' approach to introduce a circle theorem

Suggested activities

KM: Right angle challenge
KM: Thales Theorem
KM: 6 point circles, 8 point circles, 12 point circles
KM: Dynamic diagrams
NRICH: Circle theorems
Hwb: Cadair Idris
Hwb: Cyclic quadrilaterals

Learning review
GLOWMaths/JustMaths: Sample Questions Both Tiers
GLOWMaths/JustMaths: Sample Questions Higher Tiers

Possible misconceptions

- Some students may think that a cyclic quadrilateral is formed using three points on the circumference along with the centre of the circle
- Some students may not appreciate the significance of standard geometrical notation for equal lengths and angles, and think that lengths / angles are equal 'because they look equal'
- Some students may not realise that they can extend the lines on diagrams to help establish necessary facts

UNIT 17 – FURTHER ALGEBRA — GCSE EDEXCEL HIGHER TEXTBOOK

Key concepts (GCSE subject content statements)

The Big Picture: Algebra progression map

- find approximate solutions to equations numerically using iteration
- solve two linear simultaneous equations in two variables algebraically

Return to overview

Possible themes

- Find approximate solutions to complex equations
- Solve simultaneous equations
- Solve problems involving simultaneous equations

Possible key learning points

- Understand the meaning of an iterative process
- Show that a solution to a complex equation lies between two given values
- Use an iterative formula to find approximate solutions to equations
- Use an iterative formula to find approximate solutions, to a given number of decimal places, to an equation
- Solve two linear simultaneous equations in two variables by substitution
- Solve two linear simultaneous equations in two variables by elimination (multiplication of both equations required)
- Solve two linear simultaneous equations in two variables by elimination (fractional coefficients)
- Derive and solve two simultaneous equations in complex cases
- Interpret the solution to a pair of simultaneous equations

Prerequisites

- Understand the concept of solving simultaneous equations by elimination
- Solve two linear simultaneous equations in two variables in very simple cases (no multiplication required)
- Solve two linear simultaneous equations in two variables in simple cases (multiplication of one equation only required)

Mathematical language

Unknown
Solve
Solution set
Interval
Decimal search
Iteration
Simultaneous equations
Substitution
Elimination

Notation

(a, b) for an open interval
[a, b] for a closed interval

Pedagogical notes

'Interval bisection' is often an intuitive approach used by pupils when faced with a certain type of problem (see below). 'Decimal search' includes 'trial and improvement' when the equation is not set to 0.
Having been introduced to iterative processes, iteration is explained as a process for finding approximate solutions to non-linear equations. GCSE examples can be found here.
Pupils have been introduced to solving simultaneous equations using elimination in simple cases in Stage 9. This includes either no multiplication being required or multiplication of just one equation being required. Solving simultaneous equations using substitution is new to this Stage.
NCETM: Departmental workshops: Simultaneous equations
NCETM: Glossary

Common approaches

Pupils are taught to label the equations (1) and (2), and label the subsequent equations (3), (4), etc.
Pupils are taught to use the 'ANS' key on their calculators when finding an approximate solution using iteration

Reasoning opportunities and probing questions

- Show me a pair of simultaneous equations with a solution x = 4, y = -2. And another. And another …
- Convince me x + 2y = 11, 3x + 4y = 18 can be solved using substitution and using elimination. Which method is best in this case?
- Always/ Sometimes/ Never: Solving a pair of simultaneous equations using elimination is more efficient than using substitution

Suggested activities

KM: Introduce iterative processes (in this example, interval bisection) by challenging students to find your chosen number (between 1 and 1000000) when the only clue is 'bigger' or smaller' after each guess. Compare the final number of guesses with 20 (since 2^{20} is close to 1000000 and students will probably have very quickly developed a process of roughly bisecting intervals).
KM: Babylonian square roots – an introduction to iterative processes
KM: Pre-iteration
KM: Iteration
KM: Stick on the Maths: ALG2 Simultaneous linear equations
KM: Convinced?: ALG2 Simultaneous linear equations
NRICH: Matchless
AQA: Bridging Units Resource Pocket 4 (Skills builder 2 and 3)
Learning review
GLOWMaths/JustMaths: Sample Questions Both Tiers
GLOWMaths/JustMaths: Sample Questions Higher Tiers
KM: 10M4 BAM Task

Possible misconceptions

- Some pupils may not check the solution to a pair of simultaneous equations satisfy both equations
- Some pupils may not multiply all coefficients, or the constant, when multiplying an equation
- Some pupils may struggle to deal with negative numbers correctly when adding or subtracting the equations

Key concepts (GCSE subject content statements)

The Big Picture: Algebra progression map

- solve quadratic inequalities in one variable
- solve two simultaneous equations in two variables where one is quadratic algebraically

Return to overview

Possible themes	Possible key learning points
• Solve inequalities • Solve simultaneous equations **Bring on the Maths: GCSE Higher Algebra** Solving Simultaneous Equations: #4	• Solve a quadratic inequality (a = 1) • Solve a quadratic inequality (a > 1) • Solve simultaneous equations in two variables where one is a simple quadratic equation using substitution • Solve simultaneous equations in two variables where one is a more complex quadratic equation using substitution • Make connections between simultaneous equations and graphs • Solve problems involving linear and quadratic simultaneous equations

Prerequisites	Mathematical language	Pedagogical notes
• Use set notation to list a set of integers • Use a formal method to solve a linear inequality • Show a range of values that solve an inequality on a number line • Sketch a graph of a quadratic functions • Find the roots of a quadratic function • Solve two linear simultaneous equations in two variables by substitution • Solve two linear simultaneous equations in two variables by elimination (multiplication of both equations required)	Unknown (Quadratic) inequality Variable Manipulate Solve Solution set Simultaneous equations Substitution Elimination **Notation** The inequality symbols: < (less than), > (greater than), ≤ (less than or equal to), ≥ (more than or equal to)	In Stage 9, pupils have learnt about solving linear inequalities in one variable and representing the solution on a number line. In Stage 10, they learnt about solving inequalities in two variables and representing the solution set using set notation and on a graph. NCETM: Glossary **Common approaches** *All pupils should sketch the quadratic function to identify the solution set for a quadratic inequality* *All pupils should experience plotting graphs of these situations using graph-plotting software*

Reasoning opportunities and probing questions	Suggested activities	Possible misconceptions
• Show me a quadratic inequality. And another, and another, … • Kenny thinks the solution to $x^2 > 9$ is $x > 3$. Jenny thinks the solution to $x^2 > 9$ is $x < -3$. Who do you agree with? Explain your answer. • Always/Sometimes/Never: A pair of simultaneous equations in two variables where one is quadratic algebraically will have two solutions	AQA Maths: Inequalities Resourceaholic: Inequalities **Learning review** KM: 11M6 BAM Task GLOWMaths/JustMaths: Sample Questions Higher Tiers	• Some pupils may think the solution to $x^2 > 16$ is $x > 4$. • Some pupils may express the solution to a quadratic inequality using incorrect notation, e.g. $-2 > x < 2$, $-2 > x > 2$ • Some pupils may think a pair of simultaneous equations in two variables where one is quadratic will only one solution

Key concepts (GCSE subject content statements)

The Big Picture: Algebra progression map

- solve quadratic equations by completing the square and by using the quadratic formula
- deduce turning points of quadratic functions by completing the square
- deduce roots of quadratic functions algebraically
- work with general iterative processes

Return to overview

Possible themes

- Solve quadratic equations
- Solve practical problems involving quadratic equations
- Understand and use iterative processes

Bring on the Maths: GCSE Higher Algebra
Solving quadratic equations: #6, #8, #9, #10, #11, #12

Possible key learning points

- Complete the square for a quadratic expression (a = 1)
- Complete the square for a quadratic expression (a > 1)
- Solve a quadratic equation (a = 1) by completing the square
- Solve a quadratic equation (a > 1) by completing the square
- Deduce the turning point of a quadratic function by completing the square
- Deduce the roots of a quadratic function using the completed square form
- Know and apply the formula for solving a simple quadratic equation of the form $ax^2 + bx + c = 0$
- Know and apply the formula for solving more complex quadratic equation of the form $ax^2 + bx + c = 0$
- Solve equations involving fractions that can be rearranged into the form $ax^2 + bx + c = 0$
- Solve problems in probability that generate a quadratic equation
- Solve problems involving quadratic equations
- Derive an iterative formula that can be used to find approximate solutions to a complex equation

Prerequisites

- Solve a quadratic equation by rearranging and factorising
- Identify when a quadratic equation cannot be solved by factorising
- Calculate fluently with negative numbers
- Rearrange algebraic expressions and equations
- Understand and use interval bisection
- Rearrange an equation to form an iterative formula

Mathematical language

(Quadratic) equation
Factorise
Rearrange
Complete the square
Unknown
Manipulate
Maximum, minimum
Parabola
Recurrence relation
Interval bisection

Notation
The form $(x + p)^2 – q$ usually implies that completing the square is required
Recurrence relations are equations such as $x_{n+1} = 2x_n - 3$

Pedagogical notes

Problems involving quadratic equations include, for example, shapes with dimensions expressed algebraically and area given.
Students have previously explored a range of iterative processes. They should now choose an appropriate method given the situation they are faced with.

Common approaches
All pupils investigate geometric representations of completing the square
All pupils derive the formula for solving a quadratic equation by completing the square

Reasoning opportunities and probing questions

- Always / sometimes / never: a quadratic equation has two solutions (justify using values of *a*, *b* and *c*)
- Show me an example of a quadratic equation with one solution. And another, and another, …
- Explore geometric representations of completing the square. Make connections between the geometry and the algebra to make sense of the name of the process

Suggested activities

NRICH: Proof sorter – quadratic equation
NRICH: Geometric parabola

Learning review
KM: 11M2 BAM Task, 11M3 BAM Task
GLOWMaths/JustMaths: Sample Questions Higher Tiers

Possible misconceptions

- Some students may attempt to always substitute positive values for *a*, *b* and *c* when using $ax^2 + bx + c = 0$
- Some students may forget that squaring a negative number results in a positive solution
- Some students may think that $(x + p)^2 – q$ implies that *p* must be positive

UNIT 17 – FUNCTIONS	***GCSE EDEXCEL HIGHER TEXTBOOK***
Key concepts (GCSE subject content statements)	**The Big Picture**: Algebra progression map

- interpret the succession of two functions as a 'composite function'
- interpret the reverse process as the 'inverse function'

Return to overview

Possible themes	Possible key learning points
• Solve problems involving functions	• Understand the meaning of a function • Know and use the notation for composite functions • Solve problems involving composite functions • Find the inverse of a given function • Solve problems involving inverse functions

Prerequisites	Mathematical language	Pedagogical notes
• Given a function, establish outputs from given inputs • Given a function, establish inputs from given outputs • Use a mapping diagram (function machine) to represent a function • Use an expression to represent a function	Mapping Function Inverse function Composite function **Notation** $f(x)$ for a function of x $f^{-1}(x)$ for the inverse of a function, $f(x)$ $fg(x)$ for a function (f) of a function (g) of x	In Stage 7, pupils have learnt about interpreting simple expressions as functions with inputs and outputs including the use of mapping diagrams. They will not have met formal function notation such as $f(x)$, $f^{-1}(x)$ and $fg(x)$ until this unit. Some pupils may think that $fg(x)$ means 'do $f(x)$ first then $g(x)$' rather than the function $f(x)$ operating on the output of the function $g(x)$. The graph of the inverse function is the reflection of the graph of the function reflected in the line $y = x$. Note that OCR do not require students to have knowledge of function notation. NCETM: Glossary NCETM: Secondary Magazine October 2016 'It Stands to Reason' **Common approaches** *f(g(x) is interpreted as the function f(x) operating on the output of the function g(x)*

Reasoning opportunities and probing questions	Suggested activities	Possible misconceptions
• Show me function and the corresponding inverse function. And another, and another, … • Convince Kenny $g(x) = \frac{x-3}{2}$ is the inverse function of $f(x) = 2x + 3$ • Always/Sometimes/Never: $fg(x) = gf(x)$ • Find a function whose inverse is the same function	KM: Functions introduction **Learning review** GLOWMaths/JustMaths: Sample Questions Higher Tiers	• Some pupils may think $f^{-1}(x) = \frac{1}{f(x)}$ • Some pupils may think that $fg(x)$ means 'do $f(x)$ first then g(x)' • Some pupils may think that $ff(x)$ means $(f(x))^2$

UNIT 18 – VECTORS	***GCSE EDEXCEL HIGHER TEXTBOOK***
Key concepts (GCSE subject content statements)	**The Big Picture**: Position and direction progression map

- apply addition and subtraction of vectors, multiplication of vectors by a scalar, and diagrammatic and column representations of vectors

Return to overview

Possible themes	Possible key learning points
• Explore the concept of a vector • Solve problems involving vectors	• Know and use different notations for vectors, including diagrammatic representation • Add and subtract vectors • Multiply a vector by a scalar • Solve simple geometrical problems involving vectors

Prerequisites	Mathematical language	Pedagogical notes
• Understand column vector notation	Vector Scalar Constant Magnitude **Notation** ***a*** (print) and *<u>a</u>* (written) notation for vectors $\overrightarrow{AB}$ notation for vectors Column vector notation $\binom{p}{q}$, *p* = movement right and *q* = movement up	In Stage 7, pupils described a translation as a 2D vector. This unit is designed to explore vectors in more detail. Vector is a latin word for 'carrier, transporter' derived from veho ('I carry, I transport, I bear'). Vectors have magnitude and direction. Scalar is from the latin 'scala' meaning 'a flight of steps, stairs, staircase'. Scalars have magnitude but no direction. NCETM: Glossary **Common approaches** Pupils either use underline notation, such as *<u>a</u>*, or $\overrightarrow{AB}$ notation when writing vectors.

Reasoning opportunities and probing questions	Suggested activities	Possible misconceptions
• Show me a pair of values for a and b to satisfy $\binom{a}{2} + 3\binom{b}{2} = \binom{10}{8}$. And another pair. And another pair. • If $\overrightarrow{OA}$ = **<u>a</u>** and $\overrightarrow{OB}$ = **<u>b</u>**, convince me the vector $\overrightarrow{AB}$= **<u>b</u>** – **<u>a</u>** • Always/Sometimes/Never: $\overrightarrow{AB}=-\overrightarrow{BA}$	KM: Vectors NRICH: Vectors CIMT: Vectors AQA: Bridging Units: Vectors **Learning review** GLOWMaths/JustMaths: Sample Questions Both Tiers GLOWMaths/JustMaths: Sample Questions Higher Tiers KM: 10M12 BAM Task	• Some pupils may try to write column vectors as fractions, i.e. $\left(\frac{1}{2}\right)$ instead of $\binom{1}{2}$ • If $\overrightarrow{OA}$ = **<u>a</u>** and $\overrightarrow{OB}$ = **<u>b</u>**, some pupils may calculate the vector $\overrightarrow{AB}$ as **<u>a</u>** – **<u>b</u>** • Some pupils may calculate $2\binom{a}{b}$ as $\binom{2a}{b}$

UNIT 18 – FURTHER VECTORS — GCSE EDEXCEL HIGHER TEXTBOOK

Key concepts (GCSE subject content statements)

The Big Picture: Position and direction progression map

- use vectors to construct geometric arguments and proofs

Return to overview

Possible themes	Possible key learning points
• Use vectors to create geometric arguments and proofs **Bring on the Maths: GCSE Higher Shape** Organising Space: #7	• Understand how to create and present a proof involving vectors • Make deductions about situations involving vectors that are multiples of other vectors • Make deductions about situations involving vectors expressed using ratios • Make deductions about situations involving vectors and parallel lines

Prerequisites	Mathematical language	Pedagogical notes
• Understand the concept of a vector • Use diagrammatic representation of vectors • Know and use different notations for vectors • Add and subtract vectors • Multiply a vector by a scalar	Vector Scalar Constant Magnitude Collinear **Notation** ***a*** *or* ***a*** (print) and *a* (written) notation for vectors $\overrightarrow{AB}$ notation for vectors Column vector notation $\binom{p}{q}$, *p* = movement right and *q* = movement up	In Stage 10, pupils explored how addition, subtraction and multiplication is applied with vectors. This unit involves the use of vectors in geometric arguments and proofs. Vector is a latin word for 'carrier, transporter' derived from veho ('I carry, I transport, I bear'). Vectors have magnitude and direction. Scalar is from the latin 'scala' meaning 'a flight of steps, stairs, staircase'. Scalars have magnitude but no direction. NCETM: Glossary **Common approaches** *All pupils either use underline notation, such as a, or $\overrightarrow{AB}$ notation when writing vectors*

Reasoning opportunities and probing questions	Suggested activities	Possible misconceptions
• Convince me that ***a*** + ***b*** = ***b*** + ***a*** • Always/Sometimes/Never: $\overrightarrow{AB}$ = ***a*** – ***b*** ?	AQA Maths: Bridging the Gap Pocket 9: Vectors AQA Maths: Vectors **Learning review** GLOWMaths/JustMaths: Sample Questions Higher Tiers	• Some pupils may not appreciate that if a vector is a multiple of another vector, then the two vectors are parallel • Some pupils may try to write column vectors as fractions, i.e. $\left(\frac{1}{2}\right)$ instead of $\binom{1}{2}$ • If $\overrightarrow{OA}$ = ***a*** and $\overrightarrow{OB}$ = ***b***, some pupils may calculate the vector $\overrightarrow{AB}$ as ***a*** – ***b***

UNIT 19 – DIRECT & INDIRECT PROPORTION	GCSE EDEXCEL HIGHER TEXTBOOK

Key concepts (GCSE subject content statements)

The Big Picture: Ratio and Proportion progression map

- construct equations that describe direct and inverse proportion

Return to overview

Possible themes	Possible key learning points
• Explore differences between direct and inverse proportion • Solve problems involving proportion **Bring on the Maths: GCSE Higher Number** Ratio and Proportion: #4, #5	• Construct and use simple equations describing direct proportion • Construct and use more complex equations describing direct proportion • Construct and use simple equations describing inverse proportion • Construct and use more complex equations describing inverse proportion • Solve problems involving direct and inverse proportion

Prerequisites	Mathematical language	Pedagogical notes
• Recognise a graph that illustrates direct or inverse proportion • Interpret equations that describe direct or inverse proportion • Understand that X is inversely proportional to Y is equivalent to X is proportional to 1/Y • Solve problems which include finding the multiplier in a situation involving direct or inverse proportion	Direct proportion Inverse proportion Multiplier **Notation** ∝ - 'proportional to'	In Stage 9, pupils have learnt about solving problems involving direct and inverse proportion, including graphical and algebraic representations. In Stage 10, they learnt about interpreting equations that describe direct and inverse proportion. NCETM: Glossary NCETM: Department Workshop – Proportional Reasoning **Common approaches** *α is read as "proportional to"* *k is used as the 'constant of proportionality' – i.e. if y α x then y = kx*

Reasoning opportunities and probing questions	Suggested activities	Possible misconceptions
• Show me two pairs of quantities that are directly proportional. And another pair. And another pair… • Show me two pairs of quantities that are inversely proportional. And another, and another, … • Convince Kenny that 'X is inversely proportional to Y' is equivalent to 'X is proportional to 1/Y'	NRICH: Triathlon and Fitness OCR Maths: Lesson Element – Inverse Proportion AQA Maths: Ratio, Proportion and Change **Learning review** GLOWMaths/JustMaths: Sample Questions Higher Tiers	• Some pupils may think that *y* is inversely proportional to *x* means y = $\frac{x}{k}$ • Some pupils may interpret inverse proportion relationships as direct proportion • Some pupils may think that the proportionality constant has to be greater than 1.

UNIT 19 – TRANSFORMATIONS OF GRAPHS   GCSE EDEXCEL HIGHER TEXTBOOK

Key concepts (GCSE subject content statements)

The Big Picture: Algebra progression map

- recognise, sketch and interpret graphs of exponential functions y = k^x for positive values of k, and the trigonometric functions (with arguments in degrees) y = sin x, y = cos x and y = tan x for angles of any size
- sketch translations and reflections of a given function

Return to overview

Possible themes

- Explore graphs of exponential functions
- Explore graphs of trigonometric functions
- Investigate the connections between graphs of functions and their translations

Bring on the Maths: GCSE Higher Algebra
Investigating Graphs I: #9
Investigating Graphs II: #6

Possible key learning points

- Plot and use the key features of the graph of an exponential function, y = k^x, for positive values of k
- Plot and use the key features of the graph of the trigonometric function y = sin x
- Plot and use the key features of the graph of the trigonometric function y = cos x
- Plot and use the key features of the graph of the trigonometric function y = tan x
- Know the effects of transforming the graph $y = f(x)$: $f(ax)$, $af(x)$, $f(x) + a$, $f(x + a)$, $y = f(-x)$ and $y = -f(x)$
- Solve simple problems involving the transformation of graphs
- Solve more complex problems involving the transformation of graphs

Prerequisites

- Recognise, plot and interpret exponential graphs
- Plot graphs of linear, quadratic, cubic and reciprocal functions
- Find sines, cosines and tangents of given angles

Mathematical language

Exponential
Function, equation
Linear, non-linear
Quadratic, cubic, reciprocal, exponential
Parabola
Asymptote
Maximum, minimum, period
Gradient, y-intercept, x-intercept, root
Sketch, plot
Arguments

Notation
$y = mx + c$
$f(x)$, $f(ax)$, $af(x)$, $f(x) + a$, $f(x + a)$

Pedagogical notes

The use of dynamic geometry is essential for this unit, such as these Geogebra files to generate the sine, cosine and tangent graphs from the unit circle and these Autograph Activities to explore transformations of graphs.
Note the graph of $y = x^2$ is useful to explore the impact of most of the transformations but not f(-x). Some pupils find the transformation y = f(x + a) difficult to understand as they think it should be a translation $\binom{a}{0}$
Some students may ask what happens with, for example, $y = (-2)^x$. This interesting question can be explored here and here.
NCETM: Glossary

Common approaches
All teachers explain the term 'exponent' to help students understand why 'exponential' functions are called 'exponential'
All pupils should experience using dynamic software (e.g. Autograph) to explore graphs of exponential functions y = k^x for positive values of k
All pupils should experience using dynamic software (e.g. Autograph) to explore graphs of trigonometric functions (with arguments in degrees) y = sin x, y = cos x and y = tan x for angles of any size

Reasoning opportunities and probing questions

- Draw the graph of $y = 1^x$. Convince me it is an exponential function
- What's the same, what's the different: the graphs of y = sin x, y = cos x and y = tan x?
- Show me the graph of an exponential function. And another, and another, …
- Convince Kenny that the graph of f(x – 2) is a translation of the graph f(x) by $\binom{2}{0}$
- Always/Sometimes/Never: The graph of an exponential function, $y = k^x$ for positive values of k, does not intersect with the x-axis

Suggested activities

NRICH: What's that graph?
AQA Maths: Transforming Graphs
AQA Maths: Further Sketching Graphs
NRICH: Parabolic Patterns
NRICH: Tangled Trig Graphs
Don Steward: Graph Transforms

Learning review
KM: 11M5 BAM Task
GLOWMaths/JustMaths: Sample Questions Higher Tiers

Possible misconceptions

- Some pupils may think that the graphs of exponential functions $y = k^x$ for positive values of k meet or intersect the x-axis.
- Some pupils may think the graph of f(x – 2) is a translation of the graph f(x) by $\binom{-2}{0}$
- Some pupils may think the graph of f(x) + a is a translation of the graph f(x) by $\binom{a}{0}$
- Some pupils may think the graph of -f(x) a reflection of the graph f(x) in the y-axis.

UNIT 19 – RATES OF CHANGE	GCSE EDEXCEL HIGHER TEXTBOOK
Key concepts (GCSE subject content statements) • apply the concepts of average and instantaneous rate of change (gradients of chords and tangents) in numerical, algebraic and graphical contexts)	**The Big Picture**: Algebra progression map

Return to overview

Possible themes	Possible key learning points
• Manipulate quadratic functions • Solve problems involving graphs of quadratic functions • Explore rates of change **Bring on the Maths: GCSE Higher Algebra** Solving Quadratic Equations: #2, #12	• Apply the concept of average rate of change in numerical, algebraic and graphical contexts • Apply the concept of instantaneous rate of change in numerical, algebraic and graphical contexts • Solve practical problems involving rates of change

Prerequisites	Mathematical language	Pedagogical notes
• Complete the square for a given quadratic expression • Know the meaning of roots, intercepts and turning points • Identify and interpret roots, intercepts, turning points of quadratic functions graphically • Interpret the gradient at a point on a curve as the instantaneous rate of change • Know the effects of transforming the graph y = f(x): f(x) + a and f(x + a)	Function Complete the square Deduce Root Turning point, minimum, maximum Rate of change Chord Tangent Average rate of change Instantaneous rate of change **Notation** The form $(x + p)^2 – q$ usually implies that completing the square is required	The use of dynamic geometry is essential for this unit to help pupils make connections between the resulting algebraic expression from completing the square and the graph of the quadratic function. This unit provides a good opportunity to reinforce the teaching of transformations of graphs y = f(x): f(x) + a and f(x + a) as explored in the 'Stage 11 Visualising I' unit. NCETM: Glossary **Common approaches** *All pupils experience dynamic graphing software; e.g. Autograph, throughout this unit*

Reasoning opportunities and probing questions	Suggested activities	Possible misconceptions
• Convince Kenny that the co-ordinates of the turning point of $y = (x – a)^2 – b$ are (a , -b) • Always/Sometimes/Never: The y value of the co-ordinates of a turning point of a quadratic function is negative • Jenny says *'if you can't factorise a quadratic then you can't find the roots of the quadratic algebraically'*. Do you agree with Jenny? Explain your answer.	AQA Maths: Further equations and graphs AQA Maths: Sketching Graphs AQA Maths: Gradients and rate of change Resourceaholic: Quadratics Resourceaholic: Tangents and Areas **Learning review** KM: 11M4 BAM Task GLOWMaths/JustMaths: Sample Questions Higher Tiers	• Some pupils may think that the y value of the co-ordinates of a turning point of a quadratic function is always negative. • Some pupils may think the co-ordinates of the turning point of $y = (x – a)^2 – b$ are (-a , -b) • Some pupils may think the roots of the quadratic y = (x + a)(x + b) are a and b • Some students may think that $(x + p)^2 – q$ implies that p must be negative